How Far Does the Lighthouse Shine? Bunching at the Minimum Wage and Informality[*]

Lucas Finamor[†] Vinícius Nery[‡]

June 2026

Abstract

Why do so many informal workers earn the minimum wage? We argue that this phenomenon, known as the lighthouse effect, exists because the minimum wage acts as a benchmark and lowers wage-setting costs for firms. We develop a dual-sector model with monopsonistic competition and minimum wage-anchoring features that generates equilibrium wage distributions featuring bunching at the minimum wage and missing masses around it in both the formal and informal sectors. We estimate the model using Brazilian firm- and worker-level microdata. Modeling lighthouse effects matters: ignoring the informal spike overstates the effect of minimum wage increases on informal wage inequality and meaningfully alters estimated structural parameters.

Keywords: minimum wage; informality; wage inequality; wage-anchoring.

JEL Codes: D21, J31, J38, J46.

---

[*] We would like to thank Gabriel Ulyssea, Tomás Martinez, and seminar participants at FGV-EESP for their helpful comments. Nery gratefully acknowledges financial support from CAPES/CNPq and FGV-EESP. All remaining errors are our own. An initial draft of this paper was part of Nery's MSc thesis at the São Paulo School of Economics (FGV-EESP), under the title "Following the Beacon: Minimum Wage Bunching, Informality, and Wage Inequality."

[†] Sao Paulo School of Economics (FGV-EESP). Contact: lucas.finamor@fgv.br.

[‡] Sao Paulo School of Economics (FGV-EESP). Contact: vnery5@gmail.com.

1 Introduction

Minimum wages are among the most salient labor market institutions in the world, with more than 160 countries having adopted them (International Labour Organization 2024). Yet, a substantial share of the workforce in developing countries works informally and is therefore not directly subject to labor regulations (Ulyssea 2018, 2020). Despite this exclusion, a body of evidence suggests that minimum wages also shape the distribution of informal wages, a phenomenon known as the *lighthouse effect* (Neri, Gonzaga, and Camargo 2001; Lemos 2009; Boeri, Garibaldi, and Ribeiro 2011): just as a lighthouse illuminates waters far from shore, the minimum wage impacts workers beyond its legal reach.

The clearest manifestation of the lighthouse effect is the existence of a mass of informal workers earning the minimum wage, documented across several developing countries (Neumark and Corella 2021; Lombardo, Ramírez Leira, and Gasparini 2024).[1] Although a growing literature examines how minimum wages affect informal earnings more broadly (Parente, Brotherhood, and Iachan 2025; Derenoncourt et al. 2025), the bunching itself remains largely unexplored: in particular, Firpo and Portella (2025) note that no existing model rationalizes the *minimum wage spike* in the informal sector.

This paper asks two questions. First, *why* do so many informal workers earn exactly the minimum wage, despite being outside the reach of labor regulations? And second, does accounting for this bunching matter for estimating the structure of the economy and, therefore, for the effects we attribute to minimum wage policies and to other labor market interventions?

To answer these questions, we first establish three empirical facts: *(i)* there is bunching at the minimum wage for formal workers; *(ii)* there is bunching at the minimum wage for informal workers, regardless of whether they work for formal or informal firms; *(iii)* the wage distributions present a missing mass above the minimum wage, for formal and informal workers, and below the minimum wage for informal workers. These facts are persistent across time, data sources, and the identity of the reporting agent, mitigating concerns about measurement error.

To explain these facts, we argue that the minimum wage has properties that attract firms and workers in both the formal and informal sectors, as it lowers the informational and optimization requirements for wage-setting (Reyes 2024; Dube, Manning, and Naidu 2025), serves as a benchmark for fair pay (Maloney and Méndez 2004), and alleviates the conflict and wage-adjustment costs of inflation (Guerreiro et al. 2024). We show that alternative explanations cannot simultaneously account for all three facts, whereas our proposed mechanism – that the minimum wage reduces wage-setting costs – can.

We then develop a dual-sector model where formal firms face higher entry costs and must comply with labor regulations such as a binding minimum wage and payroll taxes, while informal firms avoid these costs but face distortions from size-dependent costs and enforcement. Firms

---

[1] The term "lighthouse effect" has been used to describe both bunching of informal wages at the minimum and broader spillovers across the informal wage distribution. In the rest of this paper, we reserve it to the latter.

are heterogeneous in productivity and must decide in which sector to enter based on incomplete information, following Ulyssea (2018). After entry, formal and informal firms compete for the same workers under monopsonistic competition, choosing wages to maximize profits subject to an upward-sloping labor supply curve.[2] Some firms that choose the formal sector are naturally constrained by the minimum wage, while others are attracted to it and choose to offer exactly the statutory value irrespective of the chosen sector. These features allow the model to generate non-degenerate equilibrium wage distributions in both sectors; crucially, these distributions have minimum wage spikes and missing masses, consistent with our three empirical facts.

Our model features four channels through which minimum wage increases affect the economy. First, the least productive formal incumbents exit. Second, the higher effective cost of formal entry pushes some entrants into informality, generating spillovers across the informal wage distribution. Third, intermediate-productivity formal firms previously unconstrained become bound by the new floor. Fourth, by acting as an attractor in wage-setting, firms around the old and new minimum wage adjust their wage policies.

We estimate the model via simulated method of moments using Brazilian microdata for 2003 and use it to assess the effects of the 60% increase in the real minimum wage observed between 2003 and 2012. In doing so, we consider specifications with and without lighthouse effects, which allows us to assess the relevance of modeling the informal minimum wage spike for the estimated structure of the economy and for counterfactual results. Our identification strategy explores a close link between the bunching analysis that estimates the minimum wage spike and missing masses and the parameters that govern them in the model.

Our main finding is that modeling lighthouse effects matters for the estimation of model parameters and for the implications of minimum wage policies. Not accounting for the informal spike impacts the estimated structural parameters, generating more dispersed firm productivity distributions and flatter wage supply curves, which may distort any counterfactual analysis of labor market policies. Because the spike effectively compresses the wage distribution, ignoring the mass of informal workers at the minimum wage also overstates the effect of minimum wage increases on informal wage inequality, yielding positive effects on wage dispersion that become smaller, or even reverse sign, once lighthouse effects are accounted for.

In our preferred specification, we estimate that the 60% real minimum wage increase observed between 2003 and 2012 accounts for nearly the entire 25.4% decline in formal wage

---

[2] Wage-setting power admits several microfundations: wage-posting with on-the-job-search (Burdett and Mortensen 1998), heterogeneous worker preferences over workplace amenities (Card et al. 2018), and oligopsonistic competition within a representative-household framework (Berger, Herkenhoff, and Mongey 2022, 2025). We instead use the efficiency wage approach of Rebitzer and Taylor (1995), in which unobservable effort and a size-dependent monitoring technology force larger firms to pay higher wages. We choose this approach for three reasons. First, because of the evidence that wage-posting is more prevalent in low-wage markets than bargaining (Hall and Krueger 2012; Lachowska et al. 2021). Second, it admits non-degenerate equilibrium wage distributions with mass points at interior wages, whereas on-the-job search models yield strictly continuous distributions and so cannot accommodate the bunching that motivates this paper. Third, it generates involuntary unemployment, a feature absent from most differentiated jobs models.

dispersion and for 38% of the 27.5% decline in total wage inequality. A Shapley value decomposition of these results into the channels discussed above shows that the larger bite of the policy in the formal sector accounts for the bulk of formal compression, while the attraction properties of the minimum wage decrease inequality, particularly in the informal sector. Finally, firm exit and sector switching together drive the estimated increases in informality and unemployment.

This paper adds to several strands of the literature. First, to the research that studies the effects of the minimum wage in frictional labor markets (Bontemps, Robin, and Van den Berg 2000; Van den Berg 2003; Flinn 2006; Flinn and Mullins 2021; Haanwinckel and Soares 2021; Engbom and Moser 2022; Haanwinckel 2025b; Parente, Brotherhood, and Iachan 2025; Berger, Herkenhoff, and Mongey 2025). We contribute by developing the first dual-sector model that generates bunching at the minimum wage and missing masses around it in both the formal and informal sectors.[3,4] Furthermore, the wage-anchoring mechanism extends single-sector models by generating missing masses above a binding minimum wage, a feature that standard wage-setting frameworks cannot rationalize.

Second, to the research that studies the effects of the minimum wage in labor markets with informality (Bosch and Manacorda 2010; Broecke, Forti, and Vandeweyer 2017; Pérez 2020; Neumark and Corella 2021; Lombardo, Ramírez Leira, and Gasparini 2024; Bosch et al. 2025) and particularly in Brazil (Jales 2018; Hinojosa 2019; Haanwinckel and Soares 2021; Saltiel and Urzúa 2022; Engbom and Moser 2022; Jales and Yu 2024; Haanwinckel 2025b; Parente, Brotherhood, and Iachan 2025; Derenoncourt et al. 2025). The paper most closely related to ours is Parente, Brotherhood, and Iachan (2025), who combine reduced-form evidence with a structural model to study how the minimum wage affects wage inequality in the formal and informal sectors. We contribute by developing a model that rationalizes lighthouse effects and use it to assess the effects of minimum wage increases on informality, unemployment, and wage inequality. Additionally, we show that ignoring the wage-anchoring properties of the minimum wage overstates its impacts on informal wage inequality and meaningfully affects the estimation of structural parameters, potentially impacting the conclusions of other counterfactual policies.

Third, we contribute to the literature employing behavioral frictions to study a range of economic settings (Akerlof and Yellen 1985; Chetty 2012; Kleven and Waseem 2013; DellaVigna and Gentzkow 2019; Hazell et al. 2022). In particular, we relate to the work that uses these frictions to explain why standard wage-setting models fail to account for bunching at interior points of the wage distribution absent measurement error, most notably at round numbers in Brazil (Reyes 2024) and in the US (Dube, Manning, and Naidu 2025). We contribute by studying

---

[3] Models with wage dispersion and bunching at the minimum wage in the formal sector through several mechanisms include Ramaswamy and Rowthorn (1991), Bhaskar and To (2003), Flinn (2006), Carrillo-Tudela (2009), Haanwinckel and Soares (2021), Flinn and Mullins (2021), Engbom and Moser (2022), Guo (2024), Haanwinckel (2025b), and Bosch et al. (2025). Additionally, recent research shows that formal firms anchor their wage structure to national or voluntary minimum wages, disregarding local labor market conditions (Hazell et al. 2022; Hjort, Li, and Sarsons 2025; Brochu et al. 2025; Derenoncourt and Weil 2025).

[4] While Jales (2018) and Jales and Yu (2024) account for the minimum wage spike in the informal sector in their empirical exercises, they do not microfound this behavior.

a channel for minimum wage spillovers across the wage distribution that has only been briefly discussed in these papers and generalize it beyond cases where the minimum is a round number.

Finally, we relate to a growing literature that uses monopsony power to model the labor market, either through search frictions (Burdett and Mortensen 1998; Engbom and Moser 2022); imperfect substitution of jobs through amenities (Card et al. 2018; Lamadon, Mogstad, and Setzler 2022; Azar, Berry, and Marinescu 2022; Azar et al. 2024; Parente, Brotherhood, and Iachan 2025; Haanwinckel 2025b); or oligopsonistic competition (Berger, Herkenhoff, and Mongey 2022, 2025; Hurst et al. 2022; Berger et al. 2023; Luduvice, Martinez, and Sollaci 2024; Amodio, Medina, and Morlacco 2024; Martinez and Santos 2025). We contribute by studying a microfoundation for the upward-sloping labor supply curve that differs from the search and amenity-based approaches prevalent in the literature: a combination of efficiency wages with monitoring technologies decreasing in firm size (Rebitzer and Taylor 1995; Dube, Manning, and Naidu 2018), which allows the model to rationalize the existence of both involuntary unemployment and equilibrium wage distributions with mass points.

2 Institutional Setting and Data

Minimum Wage in Brazil. Since 1984, the minimum wage in Brazil has been set in monthly gross terms by the federal government, and, in principle, applies to all regions and occupations, although in practice it extends only to formal workers.[5] Workers – formal or informal – can file claims against their employers if they are paid below the minimum. These penalties are substantial, ranging from 7.5 to 11 times the monthly minimum wage per worker depending on firm size and doubling in case of recidivism (Derenoncourt et al. 2025, Appendix A5).

The minimum wage in Brazil is adjusted annually: between 1994 and 2006 at the discretion of the federal government, and between 2007 and 2019 taking into account inflation of the past year and real GDP growth of two years prior, if positive (Schymura 2019). As a consequence, the real minimum wage rose more than 60% between 2003 and 2012, as shown in Figure A.1b. Figure A.8 shows that its ratio to the median wage – a common measure of the bite of the policy – rose in both the formal and informal sectors.

Informality. As discussed in Ulyssea (2020), "informality" may have several meanings. In this paper, we define it as the decision of firms and workers to not comply with labor regulations. Specifically, informal firms are defined as those that do not have the tax identification number required by authorities – *Cadastro Nacional da Pessoa Jurídica* (CNPJ) – and therefore neither pay taxes nor can hire workers formally. We define as formal workers those with a signed working permit (*carteira de trabalho assinada*) , which entails mandatory contributions to social

---

[5] Some higher income states (São Paulo, Rio de Janeiro, Rio Grande do Sul, Paraná and Santa Catarina) have other wage floors for specific industries, but these are often non-binding (Corseuil, Foguel, and Hecksher 2015). As Engbom and Moser (2022) and Parente, Brotherhood, and Iachan (2025), we disregard them in the analysis.

security. Informal workers, in turn, are employees without a signed working permit, and may work for formal or informal firms (Ulyssea 2018). As in Ulyssea (2018), Haanwinckel and Soares (2021), Parente, Brotherhood, and Iachan (2025), and Derenoncourt et al. (2025), we exclude the self-employed, as they have greater ability to set their own pay and generally have different characteristics from "proper" informal workers (Narita 2020; Finamor 2025).

Data Sources. Our main dataset is the *Pesquisa Nacional por Amostra de Domícilios* (PNAD), an annual, nationally representative household survey conducted by the Brazilian Institute of Geography and Statistics (*Instituto Brasileiro de Geografia e Estatística*, IBGE) with data on formal and informal workers, self-employed, employers, unemployed, and those out of the labor force. We use an alternative IBGE survey to calculate employment transitions: the *Pesquisa Mensal de Emprego* (PME), a rotating panel covering Brazil's six largest metropolitan areas.

To get data on formal firms, we use the *Relação Anual de Informações Sociais* (RAIS), an administrative dataset maintained by the Ministry of Labor covering the universe of formal firms and workers in Brazil, with both firm- and worker-level information. For informal firms, we rely on the *Pesquisa de Economia Informal Urbana* (ECINF). ECINF is a survey conducted by IBGE in 1997 and 2003 aimed at understanding small formal and informal firms. It is nationally representative for businesses with five employees or less and contains information on firms themselves, their owners – who answer the survey –, and their employees. The size restriction is not significant for informal firms, but excludes a relevant portion of formal establishments: Appendix G details how we follow Ulyssea (2018) to join RAIS and ECINF to construct a dataset representative of both formal and informal firms.[6]

Sample Restrictions. We largely follow Ulyssea (2018) and consider only urban, private-sector workers, excluding domestic activities but including construction. We restrict attention to individuals aged between 18 and 70 who work full-time (more than 40 hours a week), as the federal minimum wage is imposed at the monthly level for contracts of 44 weekly hours. We exclude the lowest and highest 1% of earnings within each sector, as well as all workers in the formal sector reporting wages below the minimum.

Descriptive Statistics. Table 1 shows some descriptive statistics of formal and informal workers and firms in Brazil in 2003, when the federal minimum wage was R$ 240 (approximately USD 78). It reveals that informality is a significant part of the economy in Brazil, considering both firms and workers.[7] Informal firms employ fewer people, while informal workers have

---

[6] Table 1 shows that 72.9% of firms in our data are informal, slightly different from the 69.8% in Ulyssea (2018, Table 4) as we consider construction activities, which he excludes. This difference is in line with Derenoncourt et al. (2025)'s findings on the prevalence of informality in the Brazilian construction sector.

[7] We understate informality by excluding self-employed, domestic, agriculture, and part-time workers, which compose a relevant share of individuals in informality. Our descriptive results on the informal minimum wage spike and on the size of the informal sector are only strengthened by the inclusion of these individuals.

lower education and receive lower wages. Despite this, there are informal workers who are college-educated and earn more than the average formal employee, while Ulyssea (2018) shows that formal and informal firms coexist within narrowly defined industries.

Similarly, Figure A.7 reveals that formal and informal firms have a sizable overlap in their productivity distributions, while Table 1 and Figures A.2–A.6 show the same for the 2003 wage distributions, irrespective of the educational group. Note that the modal wage in both sectors for all workers but the college-educated is the minimum wage. Furthermore, wage dispersion is higher in the informal sector at every educational group, so the larger dispersion and lower level of informal wages is not driven solely by educational composition. Jointly, these facts go against the "dualistic view" of informality, according to which informal establishments are systematically less productive and have significant structural differences from their formal counterparts (La Porta and Shleifer 2014).

Table 1: Sector Composition and Characteristics in 2003

	Workers							Firms	
Sector	Share (%)	P25 Wage	Median Wage	P75 Wage	Mode Wage	Var$(\log w)$	MW Spike (%)	Share (%)	Mean Size
Formal	74.9	350	490	800	240	0.44	9.5	27.1	8.0
No Middle School	31.7	300	400	600	240	0.22	14.0		
Middle School	19.3	320	450	650	240	0.26	11.2		
High School	40.3	360	500	800	240	0.38	7.0		
College	8.7	900	1500	2800	2000	0.63	0.6		
Informal	25.1	240	300	480	240	0.45	16.8	72.9	1.2
No Middle School	51.5	220	280	400	240	0.32	16.9		
Middle School	20.7	240	300	450	240	0.38	19.2		
High School	24.2	240	350	600	240	0.46	16.9		
College	3.6	650	1081	2000	1500	0.72	2.6		

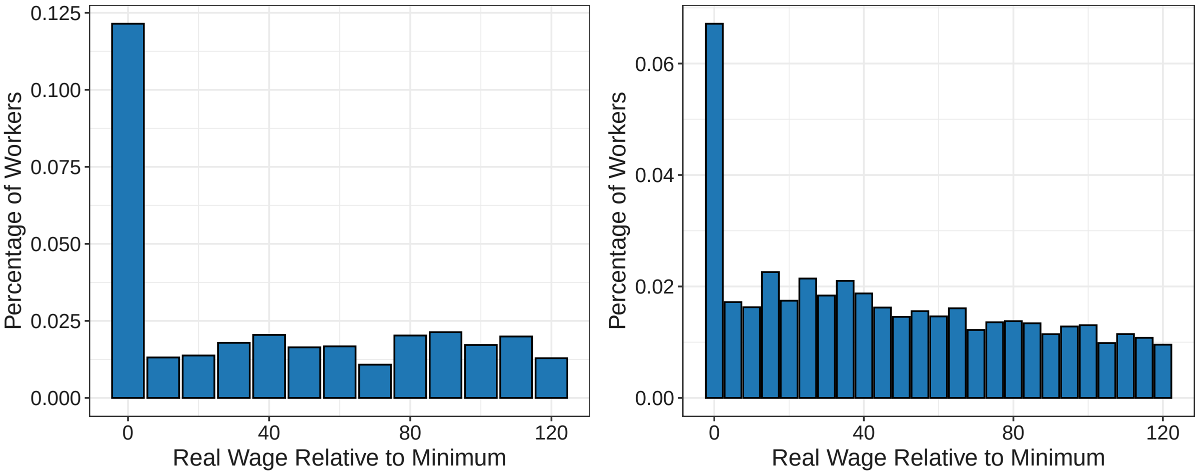
Notes: Shares of workers and firms are conditional on them being in the formal or informal sector, excluding self-employed, employers, unemployed and those out of the labor force. All statistics use survey weights. Firm-level data on both sectors is constructed by combining RAIS and ECINF following the replication code for the moments used by Ulyssea (2018). "P" denotes percentiles. Firm size is measured by the number of employed workers. Schooling shares are within-sector and sum to 100, while sector shares are relative to total employment. Minimum wage in 2003 was R$ 240 (USD 78 using the mean exchange rate of the year). Sample selection as described in Section 2.
Sources: PNAD, ECINF and RAIS (2003).

3 Empirical Facts: Minimum Wage Spikes and Missing Masses

3.1 Minimum Wage Spikes

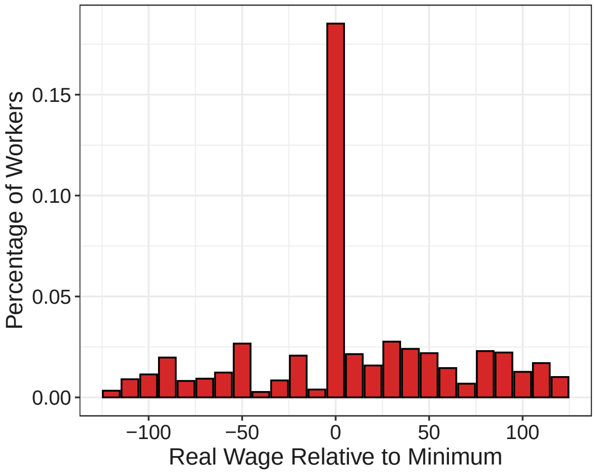
A well-documented fact in both developing and developed countries is the existence of a mass of formal workers earning the minimum wage (Lombardo, Ramírez Leira, and Gasparini 2024; Dube and Lindner 2024). Figure 1 shows the existence of a significant minimum wage spike in the distribution of formal wages in both PNAD and RAIS, where we pool wages from 2003 to 2012, express them as deviations from each year's minimum wage, and deflate them to 2003 values. Figures A.9 and A.10 show that the spike exists in all years in both sources.

A less explored fact is the existence of a minimum wage spike in the distribution of *informal* workers, who can avoid labor regulations. Known as the *lighthouse effect*, this phenomenon has been documented in Brazil (Neri, Gonzaga, and Camargo 2001; Lemos 2009; Derenoncourt



(a) Formal Sector – PNAD

(b) Formal Sector – RAIS



(c) Informal Sector – PNAD

Figure 1: Pooled Wage Distributions (2003–2012)

Notes: The figure shows the histograms of the wage distributions of 2003–2012, pooling wages across all years, expressing them as deviations from each year's minimum wage, and deflating them to 2003 values using the Consumer Price Index (IPCA). RAIS and PNAD use a binwidth of R$ 5 and R$ 10, respectively. We report habitual wages in PNAD and contractual wages in RAIS. The first and second panels are for formal workers and the third panel for informal workers. Sample selection as described in Section 2.
Sources: PNAD and RAIS (2003–2012).

et al. 2025) and in other Latin American countries (Lombardo, Ramírez Leira, and Gasparini 2024). We show the existence of the minimum wage spike in Brazil's informal sector in Figure 1c and Figure A.11, noting that the share of informal workers earning the minimum wage in PNAD is always *larger* than the respective share of formal workers. As noted by Firpo and Portella (2025), no model rationalizes the existence of the informal minimum wage spike.

A first step toward understanding this puzzle is to ask *which* informal workers earn the minimum wage. One possibility is that the spike is entirely driven by informal workers employed in formal firms, who may simply mirror the wages of their formal peers due to fairness concerns. Figure 2 tests this using ECINF, which allows us to compare informal workers across formal and informal firms.[8] Although part of the lighthouse effect can be attributed to informal workers at

---

[8] The informal spike is very similar to the one in the 2003 PNAD, while the formal spike appears larger because ECINF is only representative of firms with up to five employees. Figure A.13 shows that the share of workers

formal firms – what Ulyssea (2018) calls the intensive margin of informality –, informal workers at informal firms account for a larger share of the spike (9.3 p.p. of 16.4 p.p.) than those at formal firms (7.1 p.p.), in line with the findings of Derenoncourt et al. (2025).

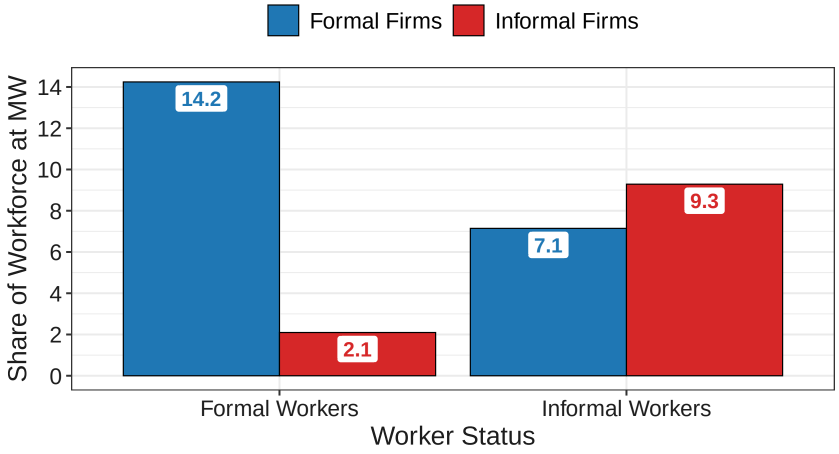


Figure 2: Minimum Wage Spike by Firm and Worker Formality Status

Notes: The figure shows the share of workers in each sector earning exactly the minimum wage decomposing by the formality status of their firm. Sample selection as described in Section 2.
Source: ECINF (2003).

Concerns About Measurement Error. Data on informal wages in Brazil are only available through self-reporting, which raises the possibility that all bunching at the minimum wage in the informal sector might be a result of measurement or reporting error in household surveys. We argue that the informal spike exists beyond these concerns.

First, Figure 1b shows a significant spike in RAIS, which contains the universe of labor contracts in Brazil. Using a different sample than ours, Derenoncourt et al. (2025, Appendix B4) shows that the formal spike in RAIS' contractual wages is nearly identical to the one in PNAD's habitual earnings in multiple years.

Second, Figure 3 shows that the informal spike exists in multiple years, always being larger than that of other sectors. Additionally, the spikes in 2005 and 2006 were the largest in our study period because the minimum wage in those years coincided with prominent round numbers (R$ 300 and R$ 350), indicating that reporting errors play a lesser role in other years where the minimum is not a prominent round number.

Third, one may think that informal workers are different from their formal counterparts in such a way that they are more likely to incorrectly report earning the minimum wage. Figure 3 provides a falsification test in that regard: the magnitude of the spike for employers and the self-employed, who have little to no reason to report exactly the minimum, is always smaller than that of formal and informal workers.[9]

---

earning the minimum wage is decreasing in firm size, while Figure A.6 shows wage histograms in ECINF.

[9] The larger spike for the self-employed relative to employers reflects the fact that they can be hired as workers (Alvarez 2024). In this sense, the smaller spike for employers further confirms PNAD's quality. In 2005, the minimum wage was R$ 300, which explains the surge in self-employed minimum wage spike.

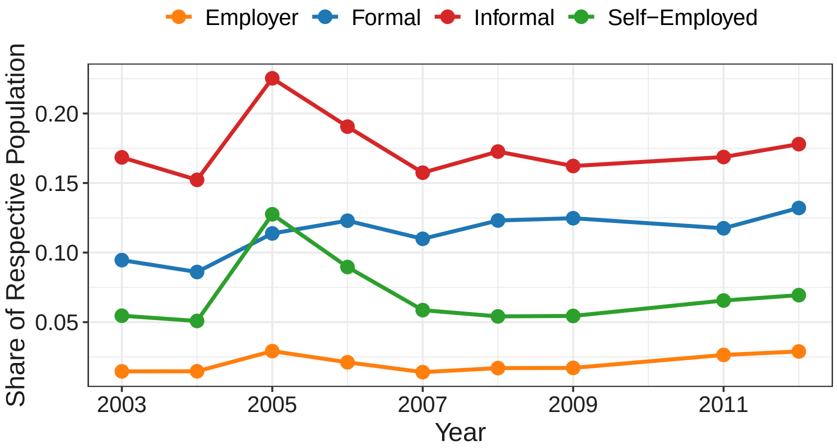


Figure 3: Minimum Wage Spike Across Years

Notes: The plot shows the share of workers in each sector who report earning exactly the minimum wage in each year (no binwidth used). Sample selection as described in Section 2.
Source: PNAD (2003–2012).

Fourth, Figure A.12a shows that the formal and informal spikes in the 2003 PNAD are larger than the mass of workers reporting the most prominent round numbers close to the minimum wage, while the opposite is true for employers and the self-employed. Finally, Figure A.12b shows that, although with slightly varying magnitudes, the informal spike exists and is larger than the formal spike across different data sources (PME, Census and ECINF). Furthermore, the existence of the spike is also robust along the margin of who reports the wage: workers (PNAD, PME, and Census) or employers (RAIS and ECINF).[10]

3.2 Missing Masses

The existence of minimum wage spikes raises a natural question: where does this mass come from? In the formal sector, standard economic theory rationalizes this spike through censoring of a latent distribution: in the absence of a minimum wage, the wage distribution would be continuous with workers earning below the statutory floor. The introduction of a minimum wage censors this lower tail, with workers either pushed up to the minimum or displaced from formal activity otherwise. Models with this mechanism include Flinn (2006), Haanwinckel and Soares (2021), Bosch et al. (2025), and Parente, Brotherhood, and Iachan (2025).

However, such truncation does not occur in the informal sector. If workers earning the minimum wage would otherwise earn nearby wages, there should be a corresponding decrease – a "missing mass" – in this area of the distribution. In the formal sector, the same logic applies above the minimum: beyond censoring of wages below the floor, the minimum wage may also attract jobs that would otherwise pay slightly higher wages. This suggests the existence of

---

[10] See Appendix C for details on wage measurement in our main datasets. As an additional robustness test, we use data from the 2000 Brazilian Census: due to the rise in the minimum wage bite over time, the spikes in this source are smaller than those in 2003. The spike in PME is lower as it only covers the largest metropolitan areas, which have higher income than the rest of the country. Conversely, the formal spike is larger in ECINF, as it is only representative of firms with up to five employees (see also Figure A.13 for the spike by number of employees).

missing masses above the minimum in both sectors and also below it in the informal sector: the histograms in Figure 1 hint at this possibility, showing slightly lower frequencies near the minimum wage.

However, what matters is not the raw histograms, but the deviations from a counterfactual distribution. Importantly, this counterfactual is not the wage distribution that would prevail absent the minimum wage entirely, but rather a "partial" counterfactual stripped only of the intensive margin responses that potentially transfer mass to the spike from its surroundings (Kleven and Waseem 2013). We follow the bunching literature and estimate these counterfactuals by fitting a smooth function to the observed wage frequencies outside a region around the minimum and measuring the excess and missing masses as deviations from this smooth distribution. Appendix B details the estimation procedure in a more general setting controlling for measurement error at round numbers ("heaping").

Let W^y be the nominal wage of a worker in year y and ${w_{\min}}^y$ be that year's minimum wage. To show the existence of missing masses, we pool the PNAD data between 2003 and 2012, express wages relative to each year's minimum ($w^y = W^y - {w_{\min}}^y$), deflate them to 2003 values, and aggregate individuals into relative wage bins of binwidth $b = 5$ in each sector (formal or informal). Letting p_w be the share of workers between 2003–2012 with relative real wage in $[w, w+b)$, we estimate the following regression with OLS:

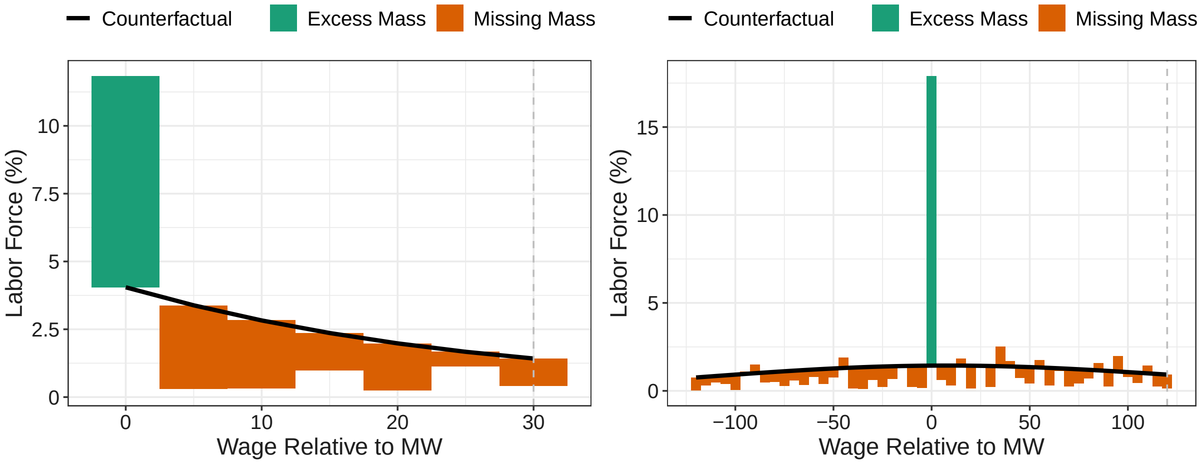
$$p_w = \sum_{j=-\Delta w}^{\Delta w} \beta_j \mathbb{1}\{w = j\} + \sum_{k=0}^{K} \alpha_k w^k + \varepsilon_w, \tag{1}$$

where $\sum_{k=0}^{K} \alpha_k w^k$ is a flexible polynomial used to approximate the counterfactual distribution and Δw is the width of the area whose mass is transferred to the bunching. β_j are the coefficients for the dummies in the excluded region: β_0 is the excess mass (EM) at the minimum wage, $\sum_{j=-\Delta w}^{-b} \beta_j$ is the missing mass below (MMB), and $\sum_{j=b}^{\Delta w} \beta_j$ is the missing mass above (MMA), where the sums run over bins $j \in \{-\Delta w, -\Delta w + b, \ldots, \Delta w\}$. Note, however, that Δw is unknown: to determine it, we follow Kleven (2016) and use an iterative procedure detailed in Appendix B that ensures that the excess mass can be accounted for by the estimated missing mass, that is, that $EM \leq MMA + MMB$. Following Dube, Manning, and Naidu (2025), we use $K = 6$ in our main specification.

Equation 1 assumes that the mass of informal workers earning the minimum wage comes from both below and above the minimum. Although we assume symmetry of the excluded range Δw above and below $w_{\min}$, the missing mass can be asymmetric, as β_j are free to vary. To be consistent with our data, we assume full compliance with the minimum wage in the formal sector, such that $MMB = 0$ and the first term becomes $\sum_{j=0}^{\Delta w} \beta_j \mathbb{1}\{w = j\}$.

Therefore, the excess mass at the minimum wage in the formal sector beyond that predicted by the polynomial comes only from above $w_{\min}$. Implicitly, this assumes that the polynomial already accounts for workers who would have earned below $w_{\min}$ absent its bite, that is, for cases where the spike may come from compliance and censoring below w_{min}. Note, however, that we *do not* assume that the entirety of the formal spike comes from the missing mass above the minimum wage. As Kleven and Waseem (2013) and Kleven (2016) argue, this bunching approach estimates a "partial counterfactual" stripped only of intensive margin responses, and not the full counterfactual that would prevail in the absence of w_{min}.

Figure 4 shows the estimated counterfactuals, excess masses at the minimum wage, and missing masses around it. Relative to these partial counterfactual frequencies, both sectors have excess masses at the minimum wage and missing masses around it, with the deficit being more prominent at wages closer to the minimum. Furthermore, even in this local bunching approach, the excess mass at the minimum wage is larger in the informal sector. In the formal sector, we estimate that wages up to R$ 30 (12.5%) above the minimum wage can be attracted to it, while the missing mass region in the informal sector is much larger in order to account for the larger excess mass.[11] These results confirm that both sectors exhibit missing masses near the minimum wage relative to the partial counterfactual distributions, indicating that the minimum absorbs jobs from nearby wage values.



(a) Formal Sector

(b) Informal Sector

Figure 4: Bunching: Excess Mass, Missing Masses and Counterfactual Distributions (2003–2012)

Notes: This figure shows the excess mass at the minimum wage (MW) using the estimated $\Delta^* w$ in each sector. Counterfactual distributions calculated using the fitted polynomial, excluding dummies. Equation 1 estimated on an interval $I = 325$ around w_{min} (or above it in the formal sector), chosen based on a local stability approach. "Excess Mass" equals the $\hat{\beta}_{w_{\text{min}}}$ coefficient. "Missing Mass" is the difference between the actual and counterfactual distributions. Details are in Appendix B. Sample selection as described in Section 2.
Source: PNAD (2003–2012).

Robustness. Table B.2 and Table B.4 show robustness exercises on the degree of the polynomial $K \in \{4,5,6,7\}$ and the binwidth $b \in \{5,10\}$, as well as standard errors calculated with a residual bootstrap detailed in Appendix B. They confirm the existence of missing masses particularly in the informal sector, and the estimated widths are robust across the polynomials with $K \in \{6,7\}$ in both binwidths. In the formal sector, the excess and missing mass are imprecisely estimated

---

[11] Reyes (2024) and Dube, Manning, and Naidu (2025) find widths of around 6–8% around prominent round numbers, which have less bunching than the minimum wage in the Brazilian setting.

and often statistically insignificant, suggesting that truncation accounts for most of the spike in this sector.

We also replace the polynomial counterfactual with a parametric alternative: a mixture of lognormals fitted to each sector via minimum distance, targeting the minimum wage spike and wage percentiles. We augment this baseline with a missing mass region, characterized by height and width parameters and identified by the difference between observed and fitted frequencies near the minimum wage, whose displaced mass is transferred to the spike. In the formal sector, we additionally censor the latent distribution at the minimum wage, generating a spike even absent missing masses. Figure B.4 and Figure B.5 confirm the existence of missing masses for both 2003 and 2003–2012 in this method, though their estimated widths are smaller than those obtained with the bunching approach.

Summary. This section documents three important empirical facts: *(i)* a minimum wage spike for formal workers; *(ii)* a minimum wage spike for informal workers beyond measurement error, irrespective of whether they are working for formal or informal firms; *(iii)* missing masses on wages around the minimum wage for both formal workers and informal workers. These facts are robust across years (2003–2012), data sources (PNAD, PME, RAIS, ECINF), identity of reporting (workers or firms), and estimation methodologies for the missing masses.

4 The Minimum Wage as a Cost-Saving Tool

The previous section outlined three stylized facts that will guide our modeling choices: minimum wage spikes in both the formal and informal sectors and missing masses around the minimum in both sectors. We propose a framework that jointly rationalizes these three facts, and show that alternative explanations cannot account for all of them on their own.

We claim that firms offer the minimum wage because it is convenient to do so. As argued by Reyes (2024) in the context of round numbers, setting the "right" wage is hard. When hiring, employers need to estimate workers' future contribution based on imperfect information about their past productivity or skills (through CVs or interviews). Firms not only need to invest time in collecting this data – time that could be invested elsewhere –, but must also form a reasonable wage estimate with this knowledge.

Instead, some employers use the minimum wage as a rule of thumb, thereby avoiding informational, administrative, time, and cognitive costs (Reyes 2024; Dube, Manning, and Naidu 2025). The minimum wage can thus serve as a simple heuristic that saves the "cost of thinking" of "thinking-averse" employers faced with a large menu of possible wage offers (Ortoleva 2013). Additionally, as the minimum wage is automatically indexed to at least inflation (see Section 2), it not only spares employers from calculating wage adjustments – which, following an efficiency wage argument, can lead to productivity decreases if they do not occur or are subpar – but also shields workers and firms from wage erosion and from the conflict costs that can arise because of it, such as tough conversations and job turnover (Guerreiro et al. 2024). Finally, the minimum is perceived as a benchmark for fair pay (Maloney and Méndez 2004), decreasing recruiting costs.

In sum, we claim that firms offer the minimum wage because it serves as a benchmark, low-cost wage. By using the minimum as a rule of thumb, firms save informational, administrative, and recruiting costs, while its indexation properties shield workers and firms from wage erosion and the conflicts that can arise from it. Note that this explanation suggests that the minimum wage can also "absorb" jobs from nearby wage values, which creates the missing masses around the minimum in both sectors.

At the wage distribution level, the minimum wage acting as a cost-saving tool for firms' wage-setting is observationally equivalent to it being a focal point in employer-employee bargaining: both can generate spikes at the minimum and missing masses around it. Note, however, that our cost-saving rationale survives under either framework: the minimum wage serving as a focal point is itself a cost-saving device, sparing firms and workers the informational, time, and cognitive costs of bargaining over a specific wage. We favor the wage-posting interpretation here and in our model because the empirical labor literature in the US documents that posting is the predominant mode of wage formation in low-wage, low-skill markets (Hall and Krueger 2012; Lachowska et al. 2021), where the minimum wage spike is most prominent (Table 1).

4.1 Alternative Explanations

We now discuss other possible explanations for the minimum wage spike in the informal sector. Although we are not able to completely rule them out with our data, we provide evidence that they do not fully explain the three stylized facts outlined before.

4.1.1 Competition Between Formal and Informal Employers

Usual arguments for lighthouse effects rely on the minimum wage acting as a signal for informal wage bargaining or on competition between formal and informal employers for the same pool of workers (Boeri, Garibaldi, and Ribeiro 2011; Neumark and Corella 2021). We argue that these explanations are inconsistent with the well-established existence of compensating differentials in developing countries: workers value formal jobs more than equally-paid informal ones – even accounting for the higher tax burden – because of higher stability, lower earnings volatility, and better insurance against unemployment.

Evidence from Brazil (Meghir, Narita, and Robin 2015; Haanwinckel and Soares 2021), Chile (Finamor 2025), and Mexico (Campos-Vázquez et al. 2026) confirms that workers would forgo substantial wages to become formal. Therefore, these explanations combined with compensating differentials imply that the informal spike should be to the *right* of the minimum wage: workers should either bargain for higher wages to compensate for the lack of benefits, or informal employers would need to offer more to compete with formal employers paying the minimum wage. Moreover, this cannot explain why there are missing masses above the statutory value in both the formal and informal sectors.

4.1.2 Workers' Preference for the Minimum Wage

One may argue that workers, rather than firms, are attracted to the minimum wage, experiencing a discontinuous utility gain when crossing its threshold, similar to a "left-digit bias". While this could explain the missing mass below the minimum wage in the informal sector, it fails to account for the missing masses above it in both sectors. Rationalizing the latter would require preferences with a discontinuous gain from earning *exactly* the minimum wage, rather than crossing its threshold. However, it would imply that workers would forgo higher wages to earn the minimum, which seems implausible.[12]

One might also argue that informal workers prefer the minimum wage because it is the lowest amount required to contribute to social security in Brazil. However, informal workers wishing to participate in the pension system can report any wage to the social security agency, so there is no institutional reason for their earnings to bunch precisely at the minimum wage.[13]

4.1.3 Intensive Margin of Informality and Fairness Concerns

Ulyssea (2018) shows that formal firms can also hire workers informally, which he calls the intensive margin of informality. It could therefore be that the informal workers who earn the minimum wage are in formal firms, who, because of fairness concerns (Dube, Giuliano, and Leonard 2019; Giupponi and Machin 2022), equate pay of formal and informal workers. However, Figure 2 shows that the informal minimum wage spike is observed in both formal and informal firms, being larger in the latter. Furthermore, it does not provide a justification for the existence of missing masses.

4.1.4 Avoid Inspections and Fines

A possibility raised by Derenoncourt et al. (2025, Section 5.3, Appendix A5) is that firms pay informal workers the minimum wage to avoid the high penalties associated with violating labor laws. In principle, all workers, formal or not, are subject to the minimum wage (adjusted

---

[12] Reyes (2024) and Dube, Manning, and Naidu (2018, 2025) study bunching of wages at round numbers in administrative data in Brazil and the United States, respectively, and similarly attribute mass points to firm behavior. They find no discontinuity in labor supply at round numbers along the wage distribution and show that firms more likely to offer round wages have worse market outcomes, making it unlikely that such firms offer round numbers to exploit workers' behavioral biases.

[13] Figure A.14a shows that there is an increase in the share of informal workers who voluntarily contribute to social security at the minimum and above it, but they still represent less than 10% of informal workers in 2003 – less than the informal minimum wage spike. At the same time, Figure A.14b documents that the minimum wage spike is decreasing in the age/experience profile in both sectors: if social security contributions were the reason for a preference for the minimum wage, we would expect this curve to be non-decreasing.

for their hours worked), and non-compliance by formal or informal firms can lead to severe fines ranging from 7.5 to 11 times the minimum wage per worker depending on the size of the firm, and doubling in case of recidivism.

The penalty for non-compliance with the minimum wage is higher than that applied to formal firms for employing informal workers. Most labor inspections occur after some form of whistle-blowing (Cardoso and Lage 2005; Brotherhood et al. 2024), so firms have the double incentive to pay their informal workers the minimum wage: to lower the chance of inspection and to decrease penalties in case of one. While likely relevant, this also does not explain the existence of missing masses above the minimum wage, especially for formal workers and firms who comply with all labor regulations.

Summary. While each of these alternative explanations likely contributes to the empirical facts we document, none of them can individually account for all three stylized facts presented in Section 3. Moreover, some of these alternatives can be encompassed within our cost-saving mechanism: avoiding inspection costs and fines, for instance, can be viewed as one of the costs that informal firms save by posting the minimum wage.

5 Model

Our goal is to rationalize the stylized facts highlighted in the previous sections: minimum wage spikes and missing masses in both sectors. To that end, we build a model where heterogeneous firms choose sectors based on incomplete information about their productivity, as in Ulyssea (2018). The formal sector has higher entry and labor costs and a binding minimum wage, while the informal sector has lower barriers to entry but size distortions via convex informality costs. Active firms choose wages to maximize profits subject to an upward-sloping labor supply curve generated by the optimal behavior of homogeneous workers.

The model generates a minimum wage spike in the formal sector through two channels. First, firms that enter the formal sector may receive an unfavorable productivity draw and find themselves constrained by the minimum wage. Second, some firms face costs of offering wages other than the minimum, capturing the cost-saving properties discussed in Section 4. Crucially, these "wage-anchoring" firms exist in both sectors, which generates bunching at the minimum wage and missing masses around it even in the informal sector. Since only a fraction of firms exhibit the anchoring behavior, the model generates wage distributions with connected support.

5.1 Firms

Time is discrete. In every period, there is an exogenous measure N of potential entrant firms which are heterogeneous in their post-entry productivity p. All firms have technologies with constant returns to scale and no complementarities between workers, producing a homogeneous good sold at a price normalized to one in a competitive market using only labor as input.

Firms can be in one of two sectors i: formal ($i = 1$) or informal ($i = 2$). For simplicity, we do not model the intensive margin of informality (Ulyssea 2018): (in)formal firms only hire (in)formal workers. However, given the constant returns to scale assumption, one may think of two firms operating in each sector as one large business operating in both, as argued by Meghir, Narita, and Robin (2015). All firms produce the same good sold at the same price: as argued by Ulyssea (2018, 2020), formal and informal firms coexist within narrowly defined industries.

Firms in sector $i = 1, 2$ offer a single wage $w \in [\underline{w}_i, \bar{w}_i] \subseteq \mathbb{R}_{++}$ to all of their employees.[14] Jobs offer no amenities, and so are characterized by their sector and wage. Crucially, heterogeneous firms face a context analogous to monopsonistic competition in the labor market in both sectors, taking as given upward-sloping labor supply curves $\ell_i(w)$, $i = 1, 2$, which we microfound in Section 5.2. To ensure that the solution to the problem of the firm exists and is unique, we assume that $\ell_i(w)$ are twice continuously differentiable and log-concave (Kline 2025); intuitively, log-concavity means that $\log \ell_i(w)$ is concave, so $\ell_i(w)$ cannot be "too convex" for $i = 1, 2$. The combination of productivity heterogeneity with monopsony power allows the model to generate wage dispersion among similar workers. To simplify the problem of the firm, we assume that firms are small relative to the market and do not engage in strategic interactions in wage-setting, taking the labor supply curves as given. As the focus of this paper is on the informal sector – predominantly composed of small businesses – this approximation comes at little cost.

Formal firms face tax rates on payroll $\tau \geq 0$; following Haanwinckel and Soares (2021) and Parente, Brotherhood, and Iachan (2025), we abstract from revenue and profit taxes. This parameter captures all the additional expenses associated with formal labor, such as social security contributions, vacation payments, and expected severance costs; for further details, see Appendix C. Additionally, they must comply with the legal minimum wage $w_{\min}$.

Informal firms avoid these costs, but are monitored by authorities and must pay a fine when caught. The expected cost of informality is $C(\ell_2)$, which is assumed to be twice continuously differentiable, strictly increasing and convex in the number of workers ℓ_2, capturing the fact that larger firms are more visible to the government and face larger fines when inspected. Additionally, they have increasing opportunity costs in the form of restricted access to business credit and public goods, resources that become more necessary as firms grow larger.

We restrict attention to steady-state environments. In particular, aggregate prices are constant and the problem of incumbent firms is equivalent to maximizing flow profits, abstracting from dynamics. Additionally, the productivity distribution of successful entrants will be the same

---

[14] Although standard in the literature (Azar and Marinescu 2024; Kline 2025), this assumption delivers the unrealistic property that all (homogeneous) workers employed at larger firms earn higher wages relative to those at smaller establishments. Haanwinckel (2025b, Appendix B.2) shows how to relax the deterministic link between firm sizes and wage premiums while keeping an average positive relationship; as his main model, we abstract from this complication since the distribution of firm sizes and its relationship with wages is not the focus of our analysis.

as that of current incumbents, ensuring stationarity of the productivity distribution.

5.1.1 Minimum Wage and Wage-Anchoring

There exist two types of firms: Type-A and Type-B. The exogenous share of Type-A firms in each sector $i = 1,2$ is given by $\sigma_i \in [0,1]$. Type-B are *wage-anchoring* firms, facing an additional cost $\kappa_i \cdot \mathbb{1}\{w \neq w_{\min}\}$, $\kappa_i > 0$, $i = 1,2$. In other words, these firms face a cost if they post a wage other than the minimum wage.[15] As discussed in Section 4, the minimum wage acts as a cost-saving tool: some firms prefer to post it because it is a reference price in the economy and minimizes informational, time, vacancy, and cognitive costs.

Following Reyes (2024) and Dube, Manning, and Naidu (2025), we do not microfound the behavior underlying these costs. We allow these costs to vary by sector to capture how the cost-saving gains of the minimum wage may be different for formal and informal firms. This can be driven by managerial capacity, incentives, or organizational structure. Importantly, it can also reflect different incentives to avoid inspection as discussed in Section 4.1. We follow Chetty (2012), Reyes (2024), and Dube, Manning, and Naidu (2025) and model these costs as a fraction of profits, with $\kappa_i \in (0,1)$, $i = 1,2$.

For analytical convenience, we assume that Type-A and Type-B firms of productivity p operating in sector $i = 1,2$ are *identical* except for κ_i. This implies that workers are indifferent between firm types (conditional on wage and sector) and that labor markets are not segmented by firm type. Thus, conditional on the sector of operation, Type-A and Type-B firms face the same labor supply curve.

5.1.2 Incumbents and Wage-Setting

We are now ready to discuss the firms' decisions. We first focus on incumbent firms, then look at how firms choose their sectors. The distribution of productivity among active firms in sector $i = 1,2$ is denoted by $\Gamma_i(p)$ and will be shown to be absolutely continuous with a connected support $[\underline{p}_i, \infty)$, $\underline{p}_i > 0$, such that all active firms have positive profits.

Type-A Firms. Firms in each sector $i = 1,2$ choose wages w to maximize their steady-state flow profits subject to the labor supply restriction, with formal firms also constrained by the minimum wage.[16] The profit functions of Type-A firms in each sector are thus:

$$\pi_1^A(p) = \max_{w \geq w_{\min}} \Big\{ [p - (1+\tau)w]\, \ell_1(w) \Big\} \qquad \pi_2^A(p) = \max_{w \geq 0} \Big\{ [p - w]\ell_2(w) - C(\ell_2(w)) \Big\} \quad (2)$$

---

[15] We focus on $w_{\min}$, but the model can easily be extended to accommodate bunching at multiples of $w_{\min}$ or other round numbers (Reyes 2024; Dube, Manning, and Naidu 2025).

[16] The absence of decreasing returns to scale means that no formal firm finds it optimal to hire below its labor supply curve at $w_{\min}$ (Berger, Herkenhoff, and Mongey 2025; Parente, Brotherhood, and Iachan 2025).

A firm offering wage w attracts $\ell_1(w)$ workers in the formal sector, incurring a total payroll tax bill of $\tau w \ell_1(w)$, and $\ell_2(w)$ workers in the informal sector, bearing informality costs $C(\ell_2(w))$. Recall that there are no strategic interactions in wage-setting and firms take the labor supply curves as given. The following proposition characterizes the optimal wage-setting behavior of Type-A firms. For ease of exposition, we focus on the case where w_{min} is binding for at least some formal firms, but the model also accommodates the setting where all Type-A firms are not constrained by the policy.

Proposition 1 (Type-A Wage-Setting, proof)**.** *The optimal behavior of Type-A firms is such that:*

1. *Conditional on productivity p and sector $i = 1,2$, the solutions to the problem of the Type-A firm $w_i^*(p)$ exist and are unique.*
2. *Informal wages solve the first-order condition and are markdowns over productivity net of the marginal informality cost. Formal wages are either constrained by w_{min} or are interior solutions that are productivity markdowns adjusted by formal labor costs.*
3. *The equilibrium wage offer distributions of Type-A firms, denoted by $F_i^A(w)$, have connected support, are absolutely continuous except at w_{min} in the formal sector, and satisfy:*

$$F_1^A(w_1^*(p)) = \begin{cases} \Gamma_1\left(p_1^{w_{min}}\right), & p \leq p_1^{w_{min}} \\ \Gamma_1(p), & p > p_1^{w_{min}} \end{cases} \quad \text{and} \quad F_2^A(w_2^*(p)) = \Gamma_2(p), \tag{3}$$

where $p_1^{w_{min}}$ is the productivity of the most productive formal Type-A firm constrained by w_{min}.

The first two parts of Proposition 1 show that, except for formal firms of lower productivity constrained by w_{min}, optimal Type-A wages are "adjusted" markdowns on the marginal productivity of labor. In the formal sector, firms price in formal labor costs, lowering wages relative to the traditional case. In the informal sector, informality costs effectively lower the marginal productivity of workers: the cost of the marginal worker in informal firms is not only their wage, but also the increased informality cost due to the larger firm size.

The final part of Proposition 1 characterizes the equilibrium wage offer distribution of Type-A firms. In the informal sector, $F_2^A(w)$ is a transformation of $\Gamma_2(p)$, similar to the offer distributions in on-the-job-search models with continuous firm productivity. In the formal sector, we have to account for the fact that there may be a mass of firms with low productivity that are constrained by the minimum wage w_{min}; outside of these firms, there exists a continuous mapping between productivity and wages. However, rationalizing the spike in the informal sector and the missing masses above w_{min} requires the existence of Type-B firms, whose wage-setting behavior we now characterize.

Type-B Firms. Given the motivation in Section 5.1.1, the profit functions of Type-B firms are:

$$\pi_i^B(w;p) = [1 - \kappa_i \cdot \mathbb{1}\{w \neq w_{\min}\}]\,\pi_i^A(w;p), \tag{4}$$

where $\pi_i^T(w;p)$ are the profits of a firm of type $T \in \{A,B\}$ of productivity p posting a wage w in sector $= 1,2$. Note that Type-B firms cannot have greater profits than their Type-A counterparts. We are now ready to state the optimal wage-setting of Type-B firms, whose wage offer CDF in each sector $i = 1,2$ we denote by $F_i^B(w)$. We denote conditional CDFs by $F_i^B(w \mid \cdot)$.

Proposition 2 (Type-B Wage-Setting, proof)**.** *There exists a threshold* $p_1^* \in \operatorname{supp}(\Gamma_1)$ *such that:*

$$F_1^B(w \mid p_1^{w_{min}} < p < p_1^*) = \mathbb{1}\{w \geq w_{min}\} = \begin{cases} 0, & w < w_{min} \\ 1, & w \geq w_{min} \end{cases}$$

and $p_1^* > p_1^{w_{min}}$*. In the informal sector, there exist thresholds* $p_{2*}, p_2^* \in \operatorname{supp}(\Gamma_2)$ *such that:*

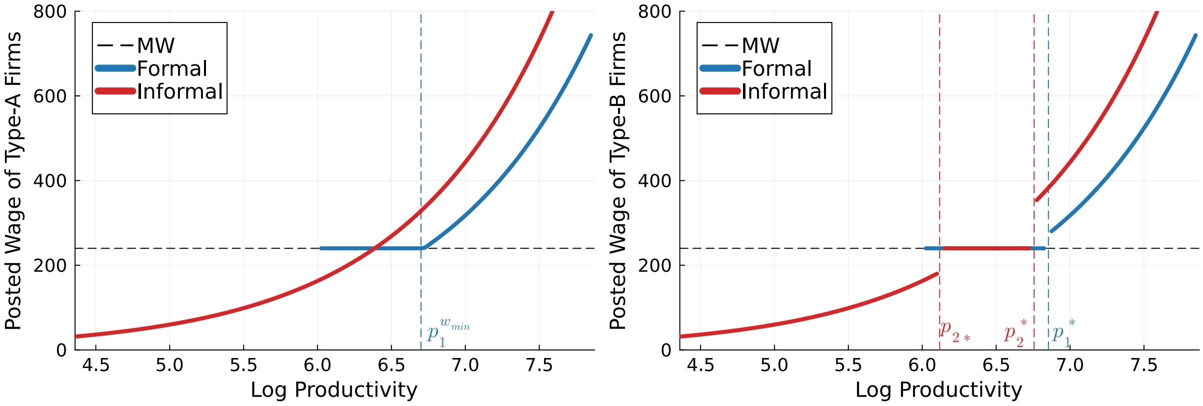
$$F_2^B(w \mid p_{2*} < p < p_2^*) = \mathbb{1}\{w \geq w_{min}\},$$

For firms with productivity below $p_1^{w_{min}}$ *and above* p_1^* *in the formal sector and below* p_{2*} *or above* p_2^* *in the informal sector, the offer distributions of Type-A and Type-B firms are the same:*

$$F_1^B(w \mid p \leq p_1^{w_{min}}) = F_1^A(w \mid p \leq p_1^{w_{min}}) \quad \textit{and} \quad F_1^B(w \mid p \geq p_1^*) = F_1^A(w \mid p \geq p_1^*)$$

$$F_2^B(w \mid p \leq p_{2*}) = F_2^A(w \mid p \leq p_{2*}) \quad \textit{and} \quad F_2^B(w \mid p \geq p_2^*) = F_2^A(w \mid p \geq p_2^*)$$

Proposition 2 shows that, except for a productivity interval in each sector, the wage offer distribution of Type-B firms is the same as that of Type-A firms. Within this productivity interval, all Type-B firms find it optimal to post $w_{\min}$, which generates a spike in the offer distribution of both sectors. Figure 5 gives visual intuition on Propositions 1 and 2. The left panel shows the relationship between productivity and wages of Type-A firms (Proposition 1): outside of the productivity interval where formal firms are constrained by $w_{\min}$ in the formal sector ($p \leq p_1^{w_{\min}}$), there exists a continuous mapping between productivity and optimal wages. The right panel shows the wages of Type-B firms (Proposition 2): there are productivity regions $p < p_1^*$ in the formal sector and $p \in (p_{2*}, p_2^*)$ in the informal sector where all Type-B firms offer the minimum wage. Outside of these regions around the minimum wage, Type-B wages are a continuous and strictly increasing function of productivity.



(a) Type-A Firms (b) Type-B Firms

Figure 5: Relationship Between Productivity and Offered Wages

Notes: The figure illustrates the relationship between productivity and optimal wages in each sector and for each type of firm. The vertical dashed lines indicate the productivity thresholds $p_1^{w_{\min}}$ (Type-A) and p_{2*}, p_2^* and p_1^* (Type-B).
Source: Own calculations based on model simulations.

5.1.3 Wage Distributions

Wage Offers. The distribution of wage offers in each sector will be given by the mixture:

$$F_i(w) = \sigma_i F_i^A(w) + (1-\sigma_i)F_i^B(w) \quad \text{for } i = 1,2, \tag{5}$$

with $F_i^A(w)$ and $F_i^B(w)$ characterized in Propositions 1 and 2, respectively.

The following proposition gives some properties on $F_i(w)$ for both sectors. For notational simplicity, it will be useful to define the interval $\mathscr{P}_i = (p_{i*}, p_i^*)$, where p_1^*, p_{2*} and p_2^* are defined as in Proposition 2 and $p_{1*} = p_1^{w_{\min}}$ for notational convenience.

Proposition 3 (Total Wage Offer Distribution, proof)**.** *The wage offer distributions $F_i(w)$ in each sector $i = 1,2$ are such that:*

1. *They have only one mass point at w_{min} and, if $\sigma_i > 0$, a connected support.*
2. *The CDF of wage offers can be written as:*

$$\begin{aligned} F_i(w_i^*(p)) &= \sigma_i F_i^A(w_i^*(p)) + (1-\sigma_i)F_i^B(w_i^*(p)) \\ &= F_i^A(w_i^*(p)) + (1-\sigma_i)\mathbb{P}(p \in \mathscr{P}_i)\left[\mathbb{1}\{w_i^*(p) \geq w_{min}\} - F_i^A(w_i^*(p) \mid p \in \mathscr{P}_i)\right] \end{aligned} \tag{6}$$

3. *For $w \neq w_{min}$, the wage offer density $f_i(w) := F_i'(w)$ is given by:*

$$\begin{aligned} f_i(w_i^*(p)) &= f_i^A(w_i^*(p)) - (1-\sigma_i)\cdot\mathbb{1}\{p \in \mathscr{P}_i\}\cdot f_i^A(w_i^*(p)) \\ &= \frac{\gamma_i(p)}{w_i^{*\prime}(p)}\left[1-(1-\sigma_i)\cdot\mathbb{1}\{p \in \mathscr{P}_i\}\right] \end{aligned} \tag{7}$$

where $f_i^A(w) := dF_i^A(w)/dw$ and $\gamma_i(p) := \Gamma_i'(p)$ denote the respective densities.

Proposition 3 shows that the total wage offer distribution in sector $i = 1,2$ is given by the offer distribution of Type-A firms plus a "distortion" created by Type-B firms. Note that, if $\sigma_i = 0$, then $f_i(w_i^*(p)) = 0$ for $p \in \mathscr{P}_i$ and $w_i^*(p) \neq w_{\min}$, highlighting the importance of Type-A firms for the claim of connected support. Additionally, the wage offer densities are no longer continuous at the boundaries of $\mathscr{P}_i$.

Bunching of Offers at the Minimum Wage. The model generates a mass of offers at $w_{\min}$ in both sectors. In the informal sector, the whole spike at the minimum wage is due to Type-B firms. In the formal sector, there are two mechanisms that create bunching at $w_{\min}$:

1. The *entry* channel discussed in Proposition 1: firms that choose to be formal may have a bad productivity draw and cannot adjust wages downward as much as they would like.
2. The *wage-anchoring* channel detailed in Proposition 2.

Let ν_i^F be the mass of firms offering $w_{\min}$ in each sector. By Propositions 1, 2 and 3:

$$\begin{aligned}
\nu_1^F &= \sigma_1 \cdot \Gamma_1\left(p_1^{w_{\min}}\right) + (1-\sigma_1) \cdot \Gamma_1\left(p_1^*\right) \\
&= \Gamma_1\left(p_1^{w_{\min}}\right) + (1-\sigma_1) \cdot \left[\Gamma_1\left(p_1^*\right) - \Gamma_1\left(p_1^{w_{\min}}\right)\right] \\
\nu_2^F &= (1-\sigma_2) \cdot \left[\Gamma_2\left(p_2^*\right) - \Gamma_2\left(p_{2*}\right)\right],
\end{aligned} \tag{8}$$

The first term in ν_1^F is the *entry* channel, capturing formal firms that are "naturally" constrained by the minimum wage because of their low productivity. The second term and the expression for ν_2^F captures the mass of offers at $w_{\min}$ which only exist due to a share $(1-\sigma_i)$ of wage-anchoring firms in each sector $i = 1,2$.[17]

Missing Masses. With a positive share of Type-B firms, Equation 7 shows that the wage density in the interval $w \in (w_i^*(p_{i*}), w_i^*(p_i^*)) \setminus \{w_{\min}\}$ is *lower* than what would prevail if $\sigma_i = 1$, which creates the *missing masses* around the minimum wage in each sector $i = 1,2$. The missing mass around $w_{\min}$ as measured by the decrease in density is thus:

$$\vartheta_i^F\left(w_i^*(p)\right) := \begin{cases} \frac{\gamma_i(p)}{w_i^{*\prime}(p)} \cdot (1-\sigma_i), & p \in \mathscr{P}_i \text{ and } w_i^*(p) \neq w_{\min} \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

$\vartheta_i^F\left(w_i^*(p)\right)$ is increasing in the share of Type-B firms $(1-\sigma_i)$, as expected. Note that, by definition, $\int_{w_i^*(p_{i*})}^{w_i^*(p_i^*)} \vartheta_i^F(w)\, dw = (1-\sigma_i)[\Gamma_i(p_i^*) - \Gamma_i(p_{i*})] = (1-\sigma_i)\mathbb{P}(p \in \mathscr{P}_i)$, which is the magnitude of the offer spike at $w_{\min}$ due to Type-B firms in each sector $i = 1,2$. This captures

---

[17] Technically, our model offers an explanation for bunching at the minimum wage in both sectors when considering *entry* wages, consistent with evidence for the US that finds that entry salaries are more likely to bunch at round numbers (Dube, Manning, and Naidu 2025). Evidence for incumbent wage-setting points to them acting to preserve within-firm wage hierarchies (Dube, Giuliano, and Leonard 2019; Brochu et al. 2025). In this direction, Cengiz et al. (2019, p. 1443) argues that the existence of minimum wage spillovers for incumbents workers may reflect "optimization frictions" in wage-setting, highlighting the role of within-firm fairness considerations.

the fact that all the extra mass at $w_{\min}$ (beyond the censoring that occurs in the formal sector) comes from the surrounding basin, generating missing masses around $w_{\min}$.

Accepted Wages. So far, we have characterized the distribution of wage *offers*. Given the form of the labor supply curves, firms offering higher wages are able to maintain a larger labor force. Thus, $F_i(w)$ are not the distributions of *accepted* wages observed in the data.

Let $G_i(w)$ be the CDF of accepted wages in each sector $i = 1,2$. These tell the proportion of workers employed in sector $i = 1,2$ earning a wage no greater than w. With this interpretation in mind, in equilibrium these are given by:

$$G_i(w) = \frac{1}{M_i} \cdot \mu_i \cdot \int_{\underline{w}_i}^{w} \ell_i(x)\, dF_i(x) \tag{10}$$

where $\underline{w}_i$ is the lowest wage offered, μ_i is the measure of active firms and $M_i = \mu_i \cdot \int_{\underline{w}_i}^{\infty} \ell_i(x) d F_i(x)$ is the total number of workers in each sector $i = 1,2$. μ_1 and μ_2 are determined in equilibrium by steady-state conditions (Section 5.1.4). Note that the spikes and missing masses in $G_i(w)$, rather than in $F_i(w)$, are the empirically relevant objects.

5.1.4 Entry and Sector Choice

Setting and Incomplete Information. We have characterized the optimal wage-setting for active firms. Now, we turn to their decision on whether and in which sector to operate. Following Haanwinckel and Soares (2021), we assume this decision is final and encompasses both the extensive and the intensive margins of informality. However, given the constant returns to scale assumption, one may think of two firms in each sector as one large firm operating in both.

Our entry process follows Ulyssea (2018), who builds on Melitz (2003). In every period, there is an exogenous measure N of potential entrants. Before entry, firms only observe a noisy signal of their true productivity, $\theta \sim H$. Intuitively, entrepreneurs can only know their true productivity after they learn the day-to-day of the business. We assume that H is absolutely continuous with $\text{supp}(H) \subseteq \mathbb{R}_{++}$, has all necessary finite moments, and is the same for all firms and periods. After entry, firms draw their actual productivity from the known conditional distribution $\Gamma(p \mid \theta)$. We assume that this distribution is the same for both sectors and independent across firms, with $\text{supp}(\Gamma) \subseteq \mathbb{R}_{++}$, absolutely continuous in p and θ and strictly decreasing in the latter, so a higher pre-entry signal θ implies a higher probability of drawing a larger p. Finally, after $p \mid \theta$ is drawn, it stays constant forever.

Regarding types, we assume that entrants know σ_i and κ_i for $i = 1,2$, but do not know their exact type, which is consistent with the noisy productivity signal and with the knowledge of $\Gamma(p \mid \theta)$. This ensures that the productivity distributions of active Type-A and Type-B firms in each sector are equal; this is done for analytical convenience and does not alter the main conclusions of the model.[18]

After entry, productivity and types are realized such that a share σ_i of entrants in each sector are of Type-A. If firms choose sector $i = 1,2$ and draw p such that $\pi_i^A(p) < 0$, they immediately exit without producing. Since there are no fixed costs of operation, this can occur only in the formal sector due to firms not being able to afford the binding minimum wage $w_{\min}$. Note that, since κ_i are percentage costs, they do not alter the sign of profits, and so the realization of types does not drive firms out of the market.

Firms' entry decisions are made with incomplete information about their productivity and types – they only observe θ, $\Gamma(p \mid \theta)$, σ_i and κ_i –, so they self-select between sectors based on *expected* profits. As Ulyssea (2018), we assume firms face an exogenous exit probability δ_i, $i = 1,2$, which is the only source of post-entry firm dynamics in our steady-state setting.[19] Given the productivity signal θ, the discounted sum of expected profits in each sector $i = 1,2$ is:

$$\begin{aligned}
\Pi_i(\theta) &= \sigma_i \cdot \Pi_i^A(\theta) + (1-\sigma_i)\cdot \Pi_i^B(\theta), \quad \text{where} \\
\Pi_i^A(\theta) &= \frac{1}{\delta_i}\int_0^\infty \max\left\{0,\, \pi_i^A(w_i^*(p);p)\right\} d\Gamma(p \mid \theta) \\
\Pi_i^B(\theta) &= \frac{1}{\delta_i}\int_0^\infty \max\Big\{0,\, (1-\kappa_i)\pi_i^A(w_i^*(p);p),\, \pi_i^A(w_{\min};p)\Big\} d\Gamma(p \mid \theta)
\end{aligned} \tag{11}$$

Entry Decisions and Productivity Overlap. We follow Ulyssea (2018) and assume firms must pay an exogenous fixed cost $E_i \in \mathbb{R}_+$ to enter sector i, such that $E_1 > E_2$, which is thereafter sunk. This reflects not only regulatory fees to enter the formal sector, but also initial investments to start production.[20] We assume E_1 and E_2 are such that entry in both sectors is positive.

Entrants' decision given their signal θ is thus summarized by the following threshold rules:

$$\begin{aligned}
\text{Enter Formal:} &\iff \Pi_1(\theta) - E_1 \geq \max\{\Pi_2(\theta) - E_2, 0\} \\
\text{Enter Informal:} &\iff \Pi_2(\theta) - E_2 > \max\{\Pi_1(\theta) - E_1, 0\} \\
\text{Stay Inactive:} &\iff \max\{\Pi_1(\theta) - E_1, \Pi_2(\theta) - E_2\} < 0
\end{aligned}$$

As the entry decision is final, the model generates overlap in the productivity distributions of both sectors, which is a salient feature of the data. Unlike Ulyssea (2018), our model has wage dispersion: since productivity maps into wages, the overlap in productivity induced by entry uncertainty helps rationalize the overlap between formal and informal wage distributions. Additionally, bunching at $w_{\min}$ arises naturally in the formal sector, as firms that draw a low $p \mid \theta$

---

[18] An alternative formulation would be to think of σ as the exogenous share of Type-A *entrants* and assume firms know their type before entry. In this setup, there would be endogenous shares σ_1 and σ_2 in each sector, and the productivity distributions of active firms would likely be different across types.

[19] These parameters can also be interpreted as sector-specific discount rates, reflecting differential access to credit lines, for example. Additionally, aggregate prices remain constant in steady-state, so the value functions have a very simple geometric series form.

[20] Since we do not have fixed costs of operation, E_1 and E_2 capture their discounted sum over the firm's life.

but still earn positive profits at $w_{\min}$ must comply with the minimum wage and cannot switch sectors or reduce wages further.

Post-Entry Productivity Distributions. Since we restrict attention to steady-state environments, the distribution of successful entrants will be the same as that of current incumbents, given by $\Gamma_1(p)$ and $\Gamma_2(p)$. Since H is absolutely continuous and entry in both sectors is positive, there exist pre-entry signals $\underline{\theta}_1$ and $\underline{\theta}_2$ such that the following "free-entry" conditions hold:

$$\Pi_2\left(\underline{\theta}_2\right) - E_2 = 0 \quad \text{and} \quad \Pi_1\left(\underline{\theta}_1\right) - E_1 = \Pi_2\left(\underline{\theta}_1\right) - E_2 \tag{12}$$

In words, $\underline{\theta}_2$ is the pre-entry signal of the firm that is indifferent between entering the informal sector and staying inactive, while $\underline{\theta}_1$ is the pre-entry signal of the marginal firm that finds entry into the formal sector profitable relative to the informal sector. Note that $E_1 > E_2$ and the assumptions on $\Gamma(p \mid \theta)$ and $C(\cdot)$ imply uniqueness of $\underline{\theta}_1$ and $\underline{\theta}_2$ and also that $\underline{\theta}_1 > \underline{\theta}_2$. Therefore, firms with $\theta \in [\underline{\theta}_2, \underline{\theta}_1)$ choose to enter the informal sector, while those with signal $\theta \geq \underline{\theta}_1$ opt to be formal. Appendix D.1 gives details on writing the post-entry productivity distributions $\Gamma_i(p)$ as a function of $\underline{\theta}_i$ and shows their continuity in each sector $i = 1, 2$.

Measure of Active Firms. In a stationary equilibrium, the endogenous measure of active firms μ_i in each sector must remain constant, which implies the following equilibrium flow conditions:

$$\mu_i \, \delta_i = N_i \quad \text{for } i = 1, 2, \tag{13}$$

where N_i denotes the measure of successful entrants in each sector. This is a steady-state condition that, given N and the thresholds $\underline{\theta}_i$ and $\underline{p}_i$, pins down the measure of active firms μ_i in each sector $i = 1, 2$. Additionally, we assume a steady-state in which the productivity distribution of entrants replicates that of exiting incumbents in each period, such that $\Gamma_1(p)$ and $\Gamma_2(p)$ are stationary. Combined with the constant measures of active firms, this guarantees that aggregate labor demand is also stationary across periods.

5.2 Workers

Our results on wage-setting rest on an upward-sloping labor supply curve, a tool used to model environments where firms have wage-setting power. It admits several microfoundations, such as wage-posting with search frictions and on-the-job-search (Burdett and Mortensen 1998); heterogeneous worker preferences over workplace amenities (Card et al. 2018); oligopsonistic competition within a representative-household framework (Berger, Herkenhoff, and Mongey 2022, 2025); and efficiency wages (Rebitzer and Taylor 1995).

For most of our analysis, including the model estimation, the reduced-form of $\ell_i(w)$ is enough. To allow for labor supply changes in counterfactuals that increase the minimum wage, we use the efficiency wage framework of Rebitzer and Taylor (1995) and Dube, Manning, and Naidu (2018, Appendix E), where effort is not perfectly observable and firms have a monitoring technology whose effectiveness decreases with firm size. Workers only produce output when exerting effort and their effort decision entails a trade-off: effort entails a utility cost, but lowers the probability of job loss. To compensate for their weaker monitoring capabilities, larger firms *must* pay higher wages to incentivize their workers to exert effort by increasing the penalty for job loss, and so a positive relationship between wages and firm sizes arises. We go into greater detail on the labor supply and on the equilibrium in Appendix D.2 and Appendix D.3, respectively.

We choose this approach for two main reasons. First, because it admits non-degenerate equilibrium wage distributions with mass points at interior wages, whereas on-the-job search models can only generate continuous distributions (Burdett and Mortensen 1998; Bontemps, Robin, and Van den Berg 2000; Piyapromdee 2018) and so cannot accommodate the bunching at the minimum wage in the informal sector that motivates this paper.[21] Second, it rationalizes involuntary unemployment, a feature absent from most differentiated jobs models.[22]

In this version of the model, workers are homogeneous. This abstraction implies that we do not address redistribution across worker types or identify which workers gain or lose most from minimum wage increases. It also sets aside firms' substitution margin between low- and high-skill labor, as well as the workforce composition channel emphasized by Haanwinckel and Soares (2021). Nevertheless, the abstraction preserves the features most relevant for our question. In particular, the model generates a non-degenerate wage distribution and is therefore well-suited to study wage inequality. Alvarez et al. (2018) document that a decline in the pass-through from firm productivity to wages – which our framework interprets as rising monopsony power – was a leading driver of the fall in formal wage inequality in Brazil between 1996 and 2012. By focusing on the firm side, our model thus captures a central mechanism behind the observed compression of the Brazilian wage distribution.

6 Identification and Estimation

We solve the model in monthly terms. To estimate its parameters, we use a two-step simulated method of moments (SMM), simulating a large number of firms across different datasets. This section describes the functional forms used for estimation, the parameters taken from the literature and from the institutional setting ("first-step"), the estimation procedure for the remaining parameters ("second-step"), and the model fit. We give further details on the estimation in Appendix G.

---

[21] Appendix E details the limitations of on-the-job search frameworks and on other mechanisms that generate mass points at the minimum wage in the formal sector. We note that, due to noncompliance in the informal sector, using the minimum as a reservation wage does not yield realistic wage distributions.

[22] Haanwinckel (2025b) has voluntary unemployment, which is also present in an extension of Parente, Brotherhood, and Iachan (2025, Appendix F).

6.1 Parametrization

To estimate the model, we must first complete its parametrization. The entry productivity signal θ follows a Pareto distribution with scale θ_{scale} and shape θ_{shape}. The effective post-entry distribution is given by:

$$p = \theta \cdot \varepsilon, \qquad \log(\varepsilon) \overset{\text{iid}}{\sim} N\left(0, \sigma_\varepsilon^2\right), \qquad \theta \perp \varepsilon, \tag{14}$$

and so $\log(p) \mid \log(\theta) \sim N(\log(\theta), \sigma_\varepsilon^2)$. ε is the post-entry shock: the higher its variance σ_ε^2, the larger the uncertainty in the firm's sector decision. This formulation makes p follow a Pareto-LogNormal distribution, which features a Pareto-like heavy tail and a LogNormal body, therefore capturing key aspects of the distributions of productivity, firm sizes, and wages (Colombi 1990; Ulyssea 2018). We note that all parameters are the same for entrants in both sectors: differences in productivity between them come solely from self-selection based on the signal θ.

The cost of informality is given by $C(\ell_2) = c\,\ell_2(w)^2$, where $c > 0$. As his model does not have wage dispersion, this simple one-parameter form is equivalent to Ulyssea (2018). Finally, the labor supply curves in each sector $i = 1, 2$ are given by:

$$\ell_i(w) = \Lambda_i \cdot w^\eta, \tag{15}$$

where η is the wage-elasticity of labor supply, which we assume is the same for both sectors. This constant elasticity formulation is common in empirical work due to its simplicity (Card et al. 2018; Lamadon, Mogstad, and Setzler 2022; Dube, Manning, and Naidu 2025; Haanwinckel 2025b; Parente, Brotherhood, and Iachan 2025). We restrict the same elasticity for both sectors as we lack a credible source of identification for separate parameters.[23]

6.2 First-Step Parameters

We set the vector of parameters $\beta_{\text{fs}} = (w_{\min}, \tau, \theta_{\text{scale}}, \delta_1, \delta_2, \Lambda_1, \Lambda_2)$ outside of the estimation routine. We set $w_{\min} = 240$ from its statutory value in the last quarter of 2003 and follow the discussion in Appendix C to set $\tau = 0.7206$. We calibrate $\theta_{\text{scale}} = w_{\min}$.[24] We take δ_1 and δ_2 from Ulyssea (2018, Table 3), converting the values from annual to monthly terms. He estimates that the exit rate of informal firms is roughly three times that of formal businesses.

---

[23] Estimates for η in the literature range anywhere from 0.1 (Dube et al. 2020) to upwards of 10 (Sokolova and Sorensen 2021). As argued by Kline (2025), there is surprisingly little work on the shape of labor supply curves; we test alternative functional forms and additional moments to identify these parameters in Section 7.1.

[24] Ulyssea (2018) and Alvarez, Pessoa, and Portela (2023) calibrate this parameter such that the minimum firm size after rounding is one, which results in a scale greater than the equilibrium wages (their models do not have wage dispersion). Because the distribution of firm sizes is not our primary concern, our choice of θ_{scale} is motivated by *(i)* the share of successful entrants in each sector relative to the number of simulated firms, which resembles the numbers in the replication code of Ulyssea (2018) and *(ii)* reasonable values for $w_2^*(\underline{\theta}_2)$, $w_2^*(\underline{\theta}_1)$, and $w_1^*(\underline{\theta}_1)$, which are the wages paid by "marginal" firms in each sector absent any productivity shocks. As in Ulyssea (2018), our entry costs and other productivity parameters are only identifiable given (and are relative to) this scale parameter.

We calibrate the labor supply constants Λ_i such that, given the estimated η, a firm offering the mean wage in the data is able to hire the mean labor force size in sector $i = 1, 2$. Formally, we calculate the constants as $\Lambda_i = \hat{\ell}_i \hat{w}_i^{-\eta}$, where $\hat{w}_i$ and $\hat{\ell}_i$ are the sample averages observed in the data in each sector $i = 1, 2$. Our wage data comes from PNAD, while the firm sizes come from the RAIS and ECINF datasets for the formal and informal sector, respectively. An alternative would be to fix the value of a constant in, say, the formal sector, and then estimate Λ_2/Λ_1. While usually the absolute scale of firm sizes does not matter, here it does because of the *(i)* costs of informality and *(ii)* entry costs, which are both additive in the value functions of firms.[25]

6.3 Second-Step Parameters

The vector of second-step parameters is $\beta_{\text{ss}} = (\eta, \sigma_1, \sigma_2, \kappa_1, \kappa_2, c, \theta_{\text{shape}}, \sigma_\varepsilon, E_1, E_2)$, which will be estimated via SMM. Although all parameters are jointly determined, we provide intuition as to how sensitive our estimates are to changes in the data given the model's structure and confirm them with the sensitivity analysis proposed by Andrews, Gentzkow, and Shapiro (2017) in Appendix I. All of our arguments are given the parameters set in the first-step.

The entry costs E_1 and E_2 are informed by the lower percentiles of the wage distribution in each sector, which includes the minimum wage spike and the missing mass above $w_{\min}$ in the formal sector. Their difference is also informed by the level of informality in the economy. Similarly, the informality cost constant c is sensitive to the share of informal workers and firms, to the wage quantiles, and to the share of workers with wages in the missing mass above $w_{\min}$ in the informal sector: a higher c increases the firm size distortion in the informal sector, so firms offer lower salaries, decreasing wages that fall in and out of the missing mass region.

The shape parameter θ_{shape} governs the dispersion of the entry productivity signal: higher values imply more concentrated distributions. Hence, it is informed by the overall shape of the wage distributions, and especially the upper quantiles of the formal sector given that $\mathbb{E}[\log(\varepsilon)] = 0$, and so formal firms paying higher wages are more likely to do so because of a higher θ. On the other hand, the variance of the post-entry shock σ_ε is more sensitive to changes in the bottom and top quantiles of informal wages: absent any shocks, there would be no informal firm paying wages below $w_2^*(\underline{\theta}_2)$ and above $w_2^*(\underline{\theta}_1)$, so informal firms that pay very low or very high wages exist only because of ε. Because of the "entry" channel in Equation 8, σ_ε is also sensitive to the formal spike: larger bunching requires a higher value of σ_ε, so more firms that choose the formal sector get a bad productivity draw and become constrained by $w_{\min}$.

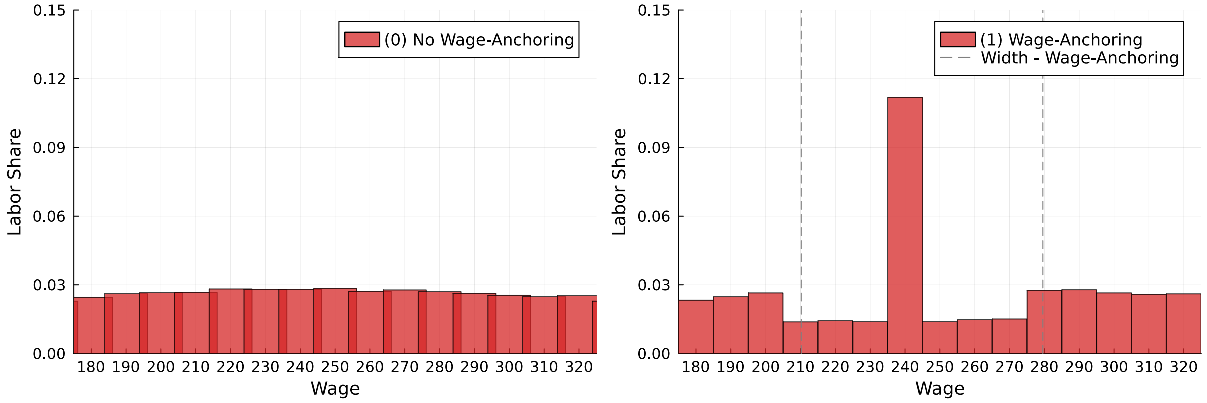
Absent general equilibrium effects, the labor supply elasticity η is governed by the wage quantiles in both sectors, as it enters the first-order conditions in Propositions 1 and 2: a higher η implies that firms cannot markdown wages as much, which increases the wage level in the

---

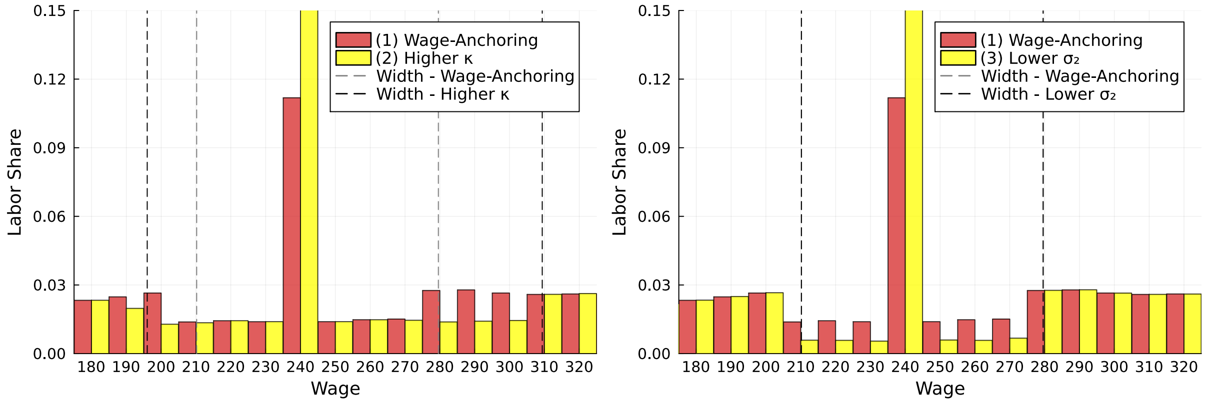
[25] Meghir, Narita, and Robin (2015, p. 1530) make a similar observation. Λ_1 and Λ_2 act as constants in the labor supply curves the same way M/N_1 and M/N_2 do in their model and, more generally, in Burdett and Mortensen (1998)-type wage-posting, on-the-job search frameworks.

economy. By the same mechanism, it is sensitive to changes in the formal minimum wage spike: everything else constant, a lower η is needed for a larger spike. Finally, it is also informed by worker informality through $\ell_1(w)$ and $\ell_2(w)$ in Equation 15.

In our model, the wage-anchoring parameters σ_i and κ_i both affect the size of the minimum wage spike and the share of workers with wages in the missing mass regions. Their identification relies on the observation that the share of Type-A firms σ_i affects the *height* of the missing mass: the smaller this share, the lower the amount of workers that earn wages just above the minimum, as more firms are of the wage-anchoring type and are attracted to w_{min}. On the other hand, κ_i determines the *width* of the missing mass: the larger the cost, the larger the wage thresholds $w_i^*(p_i^*)$. Figure 6 illustrates these mechanisms using the informal sector as an example. A larger cost κ_2 increases the width of the missing mass, measured in the model as $\Delta^* w_i(\kappa_i) = w_i^*(p_i^*) - w_{\text{min}}$, while keeping the height of the bars in the previous missing mass region constant. In contrast, a decrease in the share of Type-A firms σ_2 preserves the width while decreasing the share of workers that earn wages close to w_{min}. Both comparative statics exercises increase the informal minimum wage spike.



(a) No Wage-Anchoring (b) Wage-Anchoring



(c) Increase in κ_2 (d) Decrease in σ_2

Figure 6: Identification of Wage-Anchoring Parameters

Notes: The figure illustrates the effects of variations in κ_2 and σ_2 on the equilibrium distribution of accepted wages in the informal sector $G_2(w)$. Panels (a) and (b) show the equilibrium distribution of accepted wages in the informal sector without and with wage-anchoring, respectively. Panels (c) and (d) illustrate the effects of an increase in κ_2 and a decrease in σ_2 relative to panel (b), respectively.
Source: Own calculations based on model simulations.

While well determined in the model, the width of the missing mass is not directly observable in the data. We use techniques from the bunching literature to estimate these moments, with details given in Appendix B. As mentioned in Section 3.2, these are partial counterfactuals that abstract only from the intensive margin "attraction" properties of the minimum wage, and so match the interpretation of our model without wage-anchoring parameters. We estimate $\Delta^* w_1 = 35$ and $\Delta^* w_2 = 105$ for 2003 and conduct robustness tests in Section 7.1.

6.4 Moments and Estimation Procedure

Let $\hat{m}$ be the vector of moments computed in the data and $m(\beta_{\text{ss}}; \beta_{\text{fs}})$ be their model equivalent – both will be detailed shortly. Our estimate $\hat{\beta}_{\text{ss}}$ is given by the vector that makes the model best approximate the data in the sense of minimizing the following loss function:

$$\hat{\beta}_{\text{ss}} = \arg\min_{\beta_{\text{ss}}} \left\{ \left(m(\beta_{\text{ss}}; \beta_{\text{fs}}) - \hat{m}\right)' \times W \times \left(m(\beta_{\text{ss}}; \beta_{\text{fs}}) - \hat{m}\right) \right\}, \tag{16}$$

where W is a matrix of moment weights detailed in Appendix G.

We have 36 moments in $\hat{m}$ that inform the estimation of the 10 parameters in β_{ss}. Most of our moments come from 2003 PNAD data on workers' wages and sector composition. Specifically, we include wage quantiles in the formal and informal sector ranging from the 5th to the 80th percentile, in five percentile increments.[26] In each sector, we include the share of workers earning exactly the minimum wage – the minimum wage spikes – and the share of them that earn wages within the estimated missing mass region above w_{min}, with the width of this area as a percentage of the minimum ($\Delta^* w_i / w_{\text{min}}$) also being included as a moment. To account for firm and worker informality, we use the 2003 share of informal workers and firms presented in Table 1.[27]

We compute the equivalent vector of moments $m(\beta_{\text{ss}}; \beta_{\text{fs}})$ in the model by simulating a large number of firms across different datasets and computing the resulting equilibrium according to Definition 1. We are also interested in analyzing the sensitivity of the estimated structure of the economy to the inclusion of wage-anchoring, which rationalizes the existence of the minimum wage spike in the informal sector and of the missing masses around it in both sectors. When estimating these specifications, we remove σ_i and κ_i from β_{ss} and exclude the informal spike and the height and width of missing mass moments from $\hat{m}$ and $m(\beta_{\text{ss}}; \beta_{\text{fs}})$. This way, our estimates are comparable to models where these features are not included as targeted moments.

---

[26] To have all moments in percentage scale, we calculate the wage quantiles q in the data (call the resulting wage $\hat{w}_{i,q}$) and include as moments the share of workers earning a wage no greater than $\hat{w}_{i,q}$. To avoid giving extra weight to the minimum wage spikes, we exclude $\hat{w}_{i,q}$ that coincide with w_{min}. Finally, to deal with heaping in the PNAD data that is inconsistent with the continuous distribution of the model except at w_{min}, we smooth out the empirical CDF for all wages different than the minimum using kernel density functions; further details are in Appendix G.

[27] To better inform the informality cost c, we also tested including the share of informal workers by wage level or firm size. We found little to no improvement in fit with this specification. The same is true for using the distribution of firm sizes to inform the labor supply parameters η and Λ_i.

6.5 Estimates and Model Fit

Estimates. Table 2 shows the value of all parameters in our main estimation, alongside those from a model estimated without the wage-anchoring parameters and targeting neither the informal minimum wage spike nor moments related to the missing masses. Appendix I conducts the sensitivity analysis of Andrews, Gentzkow, and Shapiro (2017) and confirms the arguments made in Section 6.3.

In our main specification, wage-anchoring is more prevalent in the informal sector with a lower share of Type-A firms and a higher cost; the values of κ_1 and κ_2 imply that wage-anchoring firms have 1.6% and 9.5% lower profits in the formal and informal sectors, respectively.[28] We estimate a labor supply elasticity close to unity, implying that a firm offering a 1% higher wage will be able to attract approximately 0.99% more workers and indicating considerable amounts of monopsony power in the Brazilian labor market and close to the median weighted value for developing countries reported by Sokolova and Sorensen (2021, Table 1). Our estimate for the formal entry cost is far larger than the one in the informal sector.[29]

We find important differences in parameter estimates between the two specifications. Entry costs are larger in the model without wage-anchoring, while the labor supply elasticity is lower. Additionally, we estimate a lower informality cost and a more dispersed productivity distribution in the specification without lighthouse effects. As the structural estimates determine the inferred effects from counterfactual policies, these differences can meaningfully distort the analysis of any labor market policy.

Model Fit. Table 3 compares the moments generated by the models with and without wage-anchoring to those in the data. Apart from the informal spike and missing mass parameters, both specifications have a similar fit. The models accurately capture the degree of informality in the economy, although slightly underestimating firm informality and overestimating the share of informal workers. Note that we could improve this fit by explicitly estimating the labor supply constants Λ_i for $i = 1,2$, but they affect the same moments as E_1, E_2, and c, and so they are only weakly identified given our design and data. As absolute firm sizes matter, we are also not able to normalize one of these constants and estimate the other in the SMM.

Our main specification is able to closely replicate the widths of missing masses, which are entirely determined by κ_1 and κ_2 given the parameters of the productivity distribution. Figure 7 details the fit of the wage distributions, which are the main focus of our analysis.[30] Panels

---

[28] Dube, Manning, and Naidu (2025) estimate mis-optimization costs between 1% and 5% in the US, close to our formal sector estimates.

[29] There are three differences relative to Ulyssea (2018). First, we set δ_i in monthly terms instead of annual, which mechanically increases $\Pi_i(\theta)$ and requires a higher entry cost to generate the same moments. Second, our θ_{scale} is lower than his, which also requires a larger entry cost to generate the same number of active firms and to match Brazilian firm informality. Third, our entry costs also capture the discounted sum of operational fixed costs, further increasing the estimated value. Therefore, our estimates are not directly comparable to his.

[30] Note that we explicitly target the CDFs of wages as moments, not the frequency distributions.

Table 2: Parameter Estimates – With and Without Wage-Anchoring

Parameter	Description	Source or Targeted Moments	Wage-Anchoring	
			Yes	No
First Step				
w_{min}	Minimum Wage	Statutory	240	240
τ	Payroll Tax	Haanwinckel and Soares (2021)	0.72	0.72
δ_1	Formal Exit Probability	Ulyssea (2018)	0.01	0.01
δ_2	Informal Exit Probability	Ulyssea (2018)	0.03	0.03
θ_{scale}	Pareto Scale (Minimum Value)	Calibrated	240	240
Λ_1	Formal Labor Supply Constant	Calibrated	0.01	0.01
Λ_2	Informal Labor Supply Constant	Calibrated	0.00	0.00
Second Step				
η	Labor Supply Elasticity	Wage Quantiles, Formal Spike, Informality	0.99	0.67
σ_1	Formal Share of Type-A Firms	Formal Spike, Height of Formal Missing Mass	0.71	-
σ_2	Informal Share of Type-A Firms	Informal Spike, Height of Informal Missing Mass	0.64	-
κ_1	Formal Mis-Optimization Cost	Formal Spike, Width of Formal Missing Mass	0.02	-
κ_2	Informal Mis-Optimization Cost	Informal Spike, Width of Informal Missing Mass	0.10	-
c	Informality Cost Constant	Informality, Informal Wage Quantiles	76.00	56.65
θ_{shape}	Pareto Shape	Wage Quantiles, Upper Formal Quantiles	3.08	2.67
σ_ε	Std. Dev. of Post-Entry Shock	Lower and Upper Informal Quantiles, Formal Spike	0.45	0.44
E_1	Formal Entry Cost	Informality, Lower Formal Quantiles, Formal Spike	165470.1	328385.0
E_2	Informal Entry Cost	Informality, Lower Informal Quantiles	8713.5	13626.9

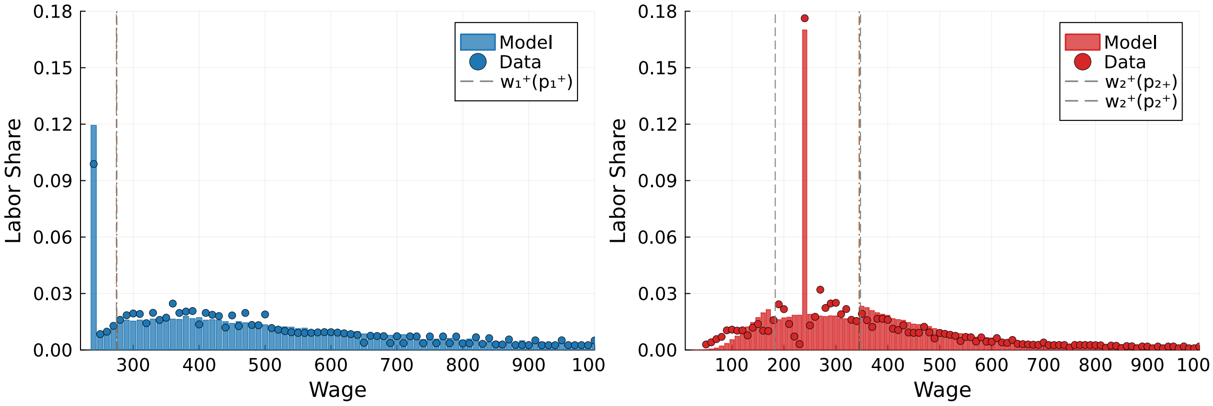
Notes: Parameters are in monthly terms and R$ 2003 values, when applicable. We display the source for the first-step parameters and targeted moments for the second-step parameters.
Source: estimation based on moments from PNAD, ECINF and RAIS (2003).

Table 3: Model Fit – With and Without Wage-Anchoring

Moment	Source	Data	Wage-Anchoring	
			Yes	No
Wage Moments				
Formal Share $w \leq 251$ (p10)	PNAD	0.100	0.120	0.110
Formal Share $w \leq 347$ (p25)	PNAD	0.251	0.258	0.260
Formal Share $w \leq 488$ (p50)	PNAD	0.501	0.482	0.487
Formal Share $w \leq 793$ (p75)	PNAD	0.750	0.759	0.758
Informal Share $w \leq 165$ (p10)	PNAD	0.101	0.084	0.098
Informal Share $w \leq 305$ (p50)	PNAD	0.502	0.490	0.469
Informal Share $w \leq 474$ (p75)	PNAD	0.750	0.785	0.784
Formal MW Spike	PNAD	0.095	0.107	0.092
Informal MW Spike	PNAD	0.168	0.151	-
Formal Share in Missing Mass Region	PNAD	0.031	0.037	-
Informal Share in Missing Mass Region	PNAD	0.211	0.185	-
Formal Width of Missing Mass	PNAD, Bunching	0.146	0.143	-
Informal Width of Missing Mass	PNAD, Bunching	0.438	0.448	-
Share of Informal Workers	PNAD	0.251	0.295	0.289
Firm Moments				
Share of Informal Firms	ECINF + RAIS	0.729	0.685	0.685

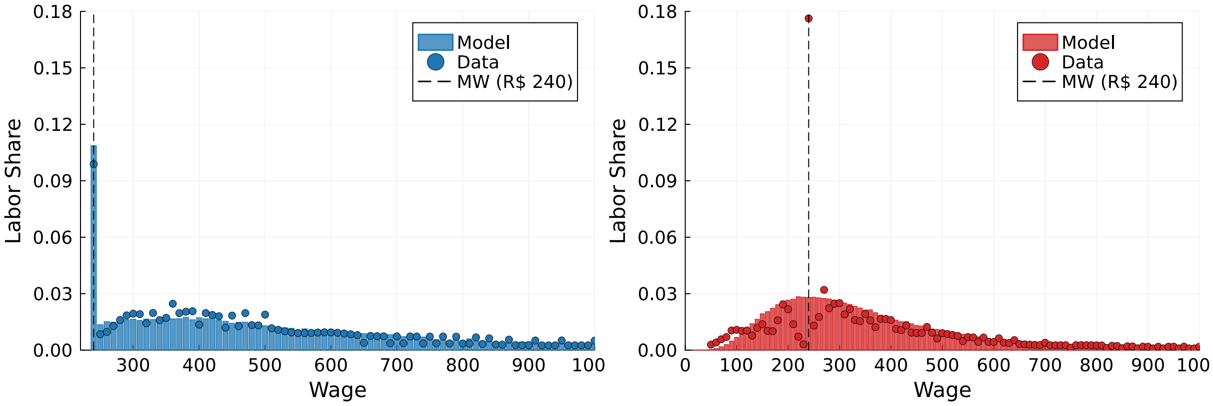
Notes: All moments are in percentage terms. Wage, spike and missing mass shares are with respect to the total of employed workers in that sector and calculated based on the distribution of accepted wages $G_i(w)$. "Share of Informal Workers" is with respect to the total of employed workers, not considering the unemployed. Width of missing masses are relative to the minimum wage.
Source: model simulations based on moments from PNAD, ECINF and RAIS (2003).

a–b show that the model with lighthouse effects can accurately capture the shape of the wage distributions and reproduce the minimum wage spike in both sectors, although overestimating it in the formal sector and slightly underestimating it in the informal sector. Panels c–d show that the specification without wage-anchoring fails to capture the informal minimum wage spike entirely, as well as the existence of missing masses above the minimum and also below it in the informal sector.



(a) Formal Sector – With Wage-Anchoring

(b) Informal Sector – With Wage-Anchoring



(c) Formal Sector – Without Wage-Anchoring

(d) Informal Sector – Without Wage-Anchoring

Figure 7: Wage Distributions: Model and Data

Notes: The figure shows the wage histograms with bins of R$ 10 in the model (bars) and those in the data (dots). Details on the construction of the empirical data are in Appendix G. In panels a–b, the gray dashed lines represent the missing mass region in the model, while the brown line is the missing mass region in the data estimated in Appendix B. In panels c–d, the black dashed line is the minimum wage.
Source: Own calculations based on model simulations and PNAD (2003).

Although we have separate wage-anchoring parameters by sector – which, in principle, could generate a near perfect match of the spikes –, there are also general equilibrium effects to consider: since Type-B firms have lower profits than Type-A counterparts, more wage-anchoring in the form of lower σ_i or higher κ_i makes sector $i = 1, 2$ less attractive, so there is a trade-off in the estimation with the informality moments. Of the 10.7% of formal workers earning the minimum wage in the model, 9.2 p.p. are in firms naturally constrained by $w_{\min}$, while the remaining 1.5 p.p. are due to wage-anchoring.

Figure H.1 and Figure H.2 give more detail on the productivity and firm size distributions, which are not targeted moments. Our main specification with wage-anchoring is able to rationalize the overlap in the productivity distributions discussed in Section 2 and, captures the shape of the firm size distributions, although generally underestimating firm sizes.

7 The Effects of Minimum Wage Increases

We now turn to estimating the effects of a minimum wage increase on the economy and their sensitivity to the existence lighthouse effects. For that, we simulate the change of 60% in the real minimum wage between 2003 and 2012 and keep all other estimated parameters fixed at the 2003 level. We re-solve the model in steady-state, and so our results should be viewed as the long-run effects of the policy.

The convenience of the parametrization of the labor supply curve $\ell_i(w)$ through Λ_i and η was that we did not have to specify all parameters of worker behavior to estimate it. However, we do require all of them to capture changes in market conditions that affect labor supply in our minimum wage counterfactuals. We detail this procedure in Appendix D.4.

Table 4 presents the effects of a 60% increase in the real minimum wage on informality, unemployment, and wage inequality, under scenarios with and without changes in market conditions (MC) and wage-anchoring. Counterfactuals that abstract from changes in market conditions rely solely on our parameterized labor supply curve, while those that take them into account require additional parameter estimates for the efficiency wage framework, which are presented Appendix D.4.

Table 4: Effects of a 60% Increase in the Minimum Wage on Informality, Unemployment, and Inequality

	Main Model			No Wage-Anchoring			Data		
	Base	Δ	Δ MC	Base	Δ	Δ MC	2003	2012	Δ
*Wage Inequality (*Var$(\log(w))$*)*									
Formal	100.0	-24.5	-23.6	100.0	-25.7	-25.6	100.0	74.6	-25.4
Informal	100.0	-1.3	-1.3	100.0	+1.6	+1.5	100.0	82.1	-17.9
Total	100.0	-10.7	-10.4	100.0	-11.3	-11.3	100.0	72.5	-27.5
Minimum Wage Spike (%)									
Formal	10.7	+22.6	+22.7	9.2	+23.0	+22.9	9.5	13.2	+3.8
Informal	15.1	+1.0	+1.0	0.0	0.0	0.0	16.8	17.8	+1.0
Total	11.9	+16.8	+16.8	6.7	+16.7	+16.7	11.3	14.0	+2.7
Informality and Unemployment (%)									
Informality (Firms)	68.5	+5.3	+5.3	68.5	+3.1	+3.1	–	–	–
Informality (Workers)	29.5	+2.6	+2.6	28.9	+1.3	+1.3	25.1	17.0	-8.1
Unemployment	25.1	+2.7	+2.4	25.1	+1.5	+1.8	25.1	14.8	-10.4

Notes: The table shows the effect of an increase of 60% in the real minimum wage $w_{\min}$. Changes are in percentage points. "MC" (market changes) considers changes in V^U discussed in Section D.4. Wage inequality measures are calculated based on the distribution of log wages normalized by their estimated baseline or 2003 values. 2012 wages deflated to R$ 2003 using the yearly consumer price index (IPCA).
Source: model simulations and PNAD (2003, 2012).

Wage Inequality. Our results agree with the large literature that finds inequality-reducing effects of the minimum wage in the formal sector: we estimate that it reduces the variance of log wages by 23.6 to 24.5%, explaining nearly the entire 25.4% empirical reduction between 2003 and 2012. Table H.1 details other inequality outcomes and shows that, as expected, the minimum wage has larger effects on measures of lower tail inequality such as the percentile ratios p_{50}/p_{10}, without affecting upper tail statistics.

Crucially, the estimated effects on informal wage inequality depend on whether the model accounts for lighthouse effects. Ignoring them overstates the impact on informal wage inequality and can lead to sign reversal: the specification with wage-anchoring yields a small decrease in the variance of log wages in the informal sector, whereas omitting it produces an increase in informal wage dispersion. The large compression of formal wages combined with small effects on informality and informal wage inequality implies a fall of 10.4%–10.7% in the total variance of log wages, with the change in the minimum wage accounting for 38% of the 27.5% fall in total wage inequality between 2003 and 2012.[31]

Informality. Focusing on the model with lighthouse effects and changes in market conditions, a higher minimum wage increases informality by 5.3 p.p. among firms and 2.6 p.p. among workers, corresponding to 7.7 and 8.8% of their baseline levels, respectively. This is driven by smaller formal firms exiting the market, while new entrants of similar productivity opt for informality due to the higher effective costs of formalization. Results are similar with and without changes in market conditions, whereas the informality response is smaller in the model without wage-anchoring, mainly because of the lower estimated elasticity η; see Figure I.4 in Appendix I for a related discussion.

Unemployment. By driving less productive formal firms out of the market and inducing new entrants to prefer the informal sector – which has smaller firms due to $C(\ell_2)$ and $\Lambda_2 < \Lambda_1$ –, the higher minimum wage raises the unemployment rate by 2.7 p.p. and 1.5 p.p. in the models without changes in market conditions, with and without wage-anchoring, respectively. Once changes in V^U are accounted for, the estimated effects are 2.4 p.p. and 1.8 p.p., respectively.

Note that the unemployment response in the model with lighthouse effects is smaller when changes in market conditions are included, as the higher minimum wage reduces V^U through lower job-finding rates ϕ and ϕ_1, an effect that dominates the boost to V^U from higher average wage offers. The resulting decline in workers' outside option allows firms to attract the same labor force at a lower wage, alleviating the unemployment margin.[32] In the model without lighthouse effects, we estimate a 0.22% increase in V^U, which increases the unemployment

---

[31] The effect on total inequality in the model without lighthouse effects is marginally larger because of the smaller informality margin of adjustment, which increases the contribution of the formal sector to the total change.

[32] With wage-anchoring, the minimum wage decreases the mass of active firms by 4.8% in the case without changes in V^U and by 4.6% when considering changes in market conditions. The possibility of passing-through the increase in the form of higher prices may make these effects smaller, as highlighted by Brochu et al. (2025).

response relative to the counterfactual without changes in market conditions.

Decomposition of Effects. The model allows us to decompose the channels behind the changes following a minimum wage increase. The *Exit* channel captures firms that succeeded in some sector in the baseline model, but fail in both sectors in the counterfactuals; without fixed costs, these are firms that still prefer the formal sector, but cannot afford the new minimum wage given their post-entry productivity. The *Switchers* channel corresponds to firms that switch sectors. The *Bite* mechanism occurs in the formal sector for firms that are "naturally constrained" by the minimum, not offering it just because they are Type-B. Finally, the *Wage-Anchoring* channel refers to Type-B firms that changed their wage policy, excluding firms that switched sectors and those already affected by the *Bite* channel. As $w_{\min}$ does not bind in the informal sector and we do not have fixed costs, the *Exit* and *Bite* channels do not affect informal firms. To quantify the contribution of each channel to distributional summary statistics, we employ a Shapley value decomposition (Shorrocks 2013), detailed in Appendix G and shown in Table 5. Given the similarity of results, we focus on the counterfactuals without changes in market conditions, as they rely on fewer parameter assumptions.

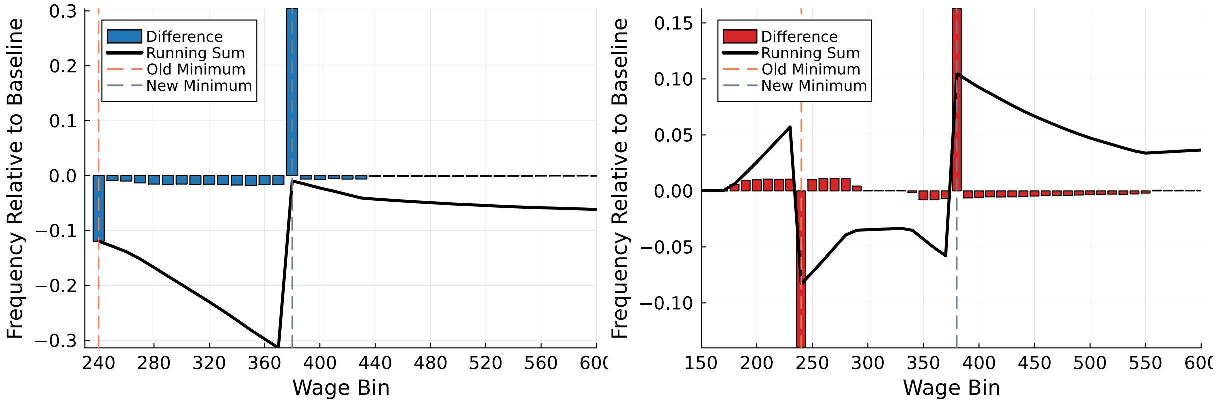
Table 5: Decomposition of Effects of a 60% Increase in the Real Minimum Wage

		MW Spike			Var(log(w))			Informality	Unemp.
Channel	Description	F	I	T	F	I	T		
Total	*Total Change*	*+22.6*	*+1.0*	*+15.8*	*−24.5*	*−1.3*	*−10.7*	*+2.6*	*+2.7*
Exit	Firms Who Became Inactive	-2.7	—	-1.9	-3.5	—	-0.5	+0.7	+1.7
Switchers	Firms Who Switched Sectors	-0.5	+0.1	-0.5	-0.3	+1.8	+2.6	+3.1	+4.7
Bite	Firms Constrained by $w_{\min}$	+23.1	—	+16.0	-20.9	—	-12.1	-1.5	-3.6
Wage-Anchoring	Type-B Firms Who Offer $w_{\min}$	+2.8	+1.0	+2.2	+0.2	-3.2	-0.7	+0.2	-0.2

Notes: The table calculates Shapley values to decompose the changes caused by an increase of 60% in the real minimum wage $w_{\min}$ into the channels discussed in the main text. "F": formal, "I": informal, "T": total economy, "Unemp.": unemployment rate. Changes are averaged using all possible permutations of channel orderings. Changes are in percentage points. "MW Spike" refers to the sum of the change in employment in the old and new minimum wage, so the change is marginally lower than that of Table 4 because of the baseline mass at the new minimum. Variance of wages are calculated based on the distribution of log wages and are normalized by their estimated baseline or 2003 values. Counterfactuals do not consider changes in V^U. Details on Shapley values are in Appendix G.
Source: model simulations.

The main driver behind changes in the spike and in wage inequality in the formal sector is the *Bite* channel: the higher minimum wage binds a larger share of firms, compressing wages. By forcing these firms to offer larger wages relative to baseline, this channel also increases firm size, alleviating the informality and unemployment effects of the policy. Firms that *Exit* and *Switch* sectors are the main drivers behind the rise in informality and unemployment. As formal firms that switch sectors are more productive in expectation than baseline informal firms, they pay higher wages on average and contribute to an increase in informal wage inequality. Wage-anchoring is key to getting the lower wage inequality in the informal sector: without it, new entrants that were previously formal would increase the variance of log wages, but the compression from Type-B firms offering the new $w_{\min}$ offsets this effect. Type-B firms also contribute to the spike in both sectors, with no significant effects on informality or unemployment.

Wage Distributions. To analyze changes along the wage distribution, we follow Cengiz et al. (2019) and Harasztosi and Lindner (2019) and plot the differences in frequencies along wage bins, normalized by the "pre-treatment" total employment in each sector. Figure 8 shows the results for counterfactuals without changes in market conditions, while Figure H.6 decomposes the result across wage bins into the channels of Table 5.



(a) Formal Sector (b) Informal Sector

Figure 8: Change in the Frequency Distribution of Wages

Notes: The figure shows the differences in R$ 10 wage bins between the new and old accepted wage distributions $G_i(w)$ following a 60% increase in the real minimum wage. Employment is normalized by the pre-treatment total labor in each sector. The black line shows the running sum of the bars until that point. Counterfactuals do not consider changes in V^U.
Source: model simulations.

The majority of the changes in the formal sector distribution occur at and below the new minimum wage: some firms that posted wages between the old and new w_{min} are constrained by it, contributing to the larger spike. At the same time, Type-B firms that offered wages above the new minimum are attracted to it due to wage-anchoring. Finally, some entrants that preferred the formal sector with the old w_{min} now switch to the informal sector. As some of these entrants may have a good draw of $p \mid \theta$, there are small negative changes above the new w_{min} and along the right tail of the distribution of formal wages. We note the visual similarity with the empirical results of Derenoncourt et al. (2025, Figure 5a).

The changes in the informal sector are heavily influenced by wage-anchoring. Firms at and around the old minimum wage adjust their schedules, while firms around the new w_{min} are attracted to it. Due to switchers from the formal sector, there is a gain in the cumulative frequency. Figure H.5 shows the changes in the specification without wage-anchoring: while the changes in the formal distribution are largely the same, the existence of lighthouse effects has profound implications for the effects along the distribution of informal wages.

Comparison of Results With Literature. Table 6 compares our main results with other papers that study the effects of the minimum wage on wage inequality and informality in Brazil. We focus on recent papers with more interpretable units, excluding, for instance, the "effective minimum wage" designs of Hinojosa (2019) and Saltiel and Urzúa (2022). To facilitate comparison across studies with different sample periods and minimum wage variations, we compute implied minimum wage elasticities as the ratio of the percentage change in each outcome to the percentage change in the real minimum wage. For a complementary summary of the literature, see Haanwinckel (2025a, Table 8).

The closest paper to ours is Parente, Brotherhood, and Iachan (2025); Appendix F provides a detailed comparison of our respective models and results. Unlike them, our model with lighthouse effects does not predict that the minimum wage increases inequality in the informal sector or in the economy as a whole. Part of the difference on total wage inequality comes from the estimated effects on informality, as they estimate an effect of 40 p.p. (105.3% of the baseline level) when analyzing the 105% increase in the minimum wage between 1996 and 2012. Our results point to a smaller informality effect, providing a middle ground between the results of Haanwinckel and Soares (2021) and Derenoncourt et al. (2025).[33] Our unemployment results are smaller than those estimated by Haanwinckel and Soares (2021) – especially considering changes in market conditions –, but larger than the formal employment effects found by Engbom and Moser (2022); our results on formal wage inequality also lie between both papers.

---

[33] Our results are not directly comparable to the headline values of Jales (2018) and Jales and Yu (2024), which compare economies with and without the minimum wage. Hence, while our estimates correspond to *marginal* effects of the minimum wage, theirs correspond to *level* impacts. Using PNAD data on 2001–2009 (Jales 2018) and 2001–2005 (Jales and Yu 2024), they find that the informal sector is 39% and 27% bigger, respectively, relative to what would prevail in the absence of the minimum wage.

Table 6: Comparison of Results and Non-Comprehensive Summary of the Literature of Minimum Wage Effects in Brazil

					Informality		Unemployment		Formal Ineq.		Informal Ineq.		Total Ineq.	
Paper	Data and Period	Method	MW Increase	Sector	Effect	Elast.	Effect	Elast.	Effect	Elast.	Effect	Elast.	Effect	Elast.
Engbom and Moser (2022)	RAIS 1996–2018	Structural	57.7%[a]	Formal	—	—	1.27%[1]	0.022	−45.3%	−0.785	—	—	—	—
Haanwinckel (2025b)	RAIS 1998–2012	Structural	93.7%	Formal	—	—	—[c]	—	−13.0%	−0.139	—	—	—	—
Haanwinckel and Soares (2021)	PME 2003–2012	Structural	62%[b]	Both	+37.5%[2]	0.605	+30.4%[3]	0.490	—	—	—	—	—	—
Parente, Brotherhood, and Iachan (2025)	PNAD 1996–2012	Structural	105%	Both	+105.3%[2]	1.003	—[d]	—	−11.5%	−0.110	+12.9%	0.123	+4.1%	0.039
Derenoncourt et al. (2025)	PNAD 1999–2009	Bunching DiD	70.9%	Both	+4.5%[2]	0.063	—	—	—	—	—	—	—	—
This paper – Main	PNAD 2003–2012	Structural	60%[b]	Both	+8.8%[2]	0.147	+10.8%[3]	0.179	−24.5%	−0.408	−1.3%	−0.022	−10.7%	−0.178
This paper – Main (MC)	PNAD 2003–2012	Structural	60%[b]	Both	+8.8%[2]	0.147	+9.6%[3]	0.159	−23.6%	−0.393	−1.3%	−0.022	−10.4%	−0.173
This paper – No Wage-Anchoring	PNAD 2003–2012	Structural	60%[b]	Both	+4.5%[2]	0.075	+6.0%[3]	0.100	−25.7%	−0.428	+1.6%	0.027	−11.3%	−0.188
This paper – No Wage-Anchoring (MC)	PNAD 2003–2012	Structural	60%[b]	Both	+4.5%[2]	0.075	+7.2%[3]	0.120	−25.6%	−0.427	+1.5%	0.025	−11.3%	−0.188

Notes: Papers may use several methods and data sources; we report the approach we use to extract the results. "Both" refer to the formal and informal sectors, excluding self-employed. The "Effect" column reports the estimate as presented in each paper, while "Elast." is the implied elasticity computed using the real minimum wage variation reported in the "MW Increase" column. The Unemployment column merges unemployment and formal employment effects. Inequality effects refer to changes in the variance of log wages. We do not report the estimates of Jales (2018) and Jales and Yu (2024) because they consider level effects instead of marginal changes in the minimum wage policy.

Effect superscripts:

[1] Formal employment: −0.007 p.p. from baseline of 54.9% for Engbom and Moser (2022, Table 7).

[2] Informality rate in percentage points: +11.01 p.p. (base 29.3%) for Haanwinckel and Soares (2021, Table A5); +40 p.p. (base 38%) for Parente, Brotherhood, and Iachan (2025, Table 5); +2.1 p.p. (base 46%) for Derenoncourt et al. (2025, Figure 6, Section 5.3, and p.7 for baseline informality); +2.6 p.p. (base 29.5%) for this paper (Main and Main MC); +1.3 p.p. (base 28.9%) for this paper (No Wage-Anchoring and No Wage-Anchoring MC).

[3] Unemployment: +4.07 p.p. from baseline of 13.4% for Haanwinckel and Soares (2021, Table A5); +2.7 p.p. and +2.4 p.p. (base 25.1%) for this paper (Main and Main MC); +1.5 p.p. and +1.8 p.p. (base 25.1%) for this paper (No Wage-Anchoring and No Wage-Anchoring MC).

MW Increase superscripts:

[a] Productivity-adjusted real MW increase, approximated by change in log points.

[b] We consider the real change between the end of 2003 and the end of 2012, hence the small differences despite the same study period.

[c] Does not report an overall employment effect but finds significant effects for lower-skilled workers; Haanwinckel (2025a) reports a 3.3 p.p. effect based on an older version.

[d] Main model does not have unemployment, but includes a version with this margin in Appendix F.

Validation: Productivity Changes. We note that, while informality responses are rather moderate in our counterfactuals, the formal minimum wage spike increases substantially: a significant number of firms still prefer to enter the formal sector and be constrained by $w_{\min}$ rather than switch to the informal sector. Given this, one may worry that our model underestimates the informality change after a minimum wage increase in favor of an unreasonably high variation in the formal spike.[34]

To alleviate these concerns, we assess whether the model can reproduce the observed change in the formal spike under plausible assumptions about productivity growth. In our model, changes in the firm productivity parameter capture several mechanisms that affected the Brazilian economy, such as gains in productivity itself, better worker composition, and more favorable terms of trade. Specifically, we take the counterfactual economy after the 60% minimum wage increase, fix firms' entry decisions, and then scale their productivity signal θ by $(1+x)$. This exercise allows us to isolate the effect of productivity on incumbent firms' wage-setting, as higher productivity allows more formal firms to pay above $w_{\min}$, reducing the spike. We then compare the change in the formal spike as a function of the productivity increase to the empirical variation between 2003 and 2012.

Figure 9 shows that the model can reproduce the empirical change in the formal spike between 2003 and 2012 with a 31% increase in firm productivity, close to the values estimated by Ferreira and Veloso (2013) and Haanwinckel and Soares (2021).[35] Hence, our model is able to reproduce the empirical change in the formal spike between 2003 and 2012 given reasonable productivity changes, which reassures our results and illustrates the arguments in Haanwinckel and Soares (2021) on the importance of considering interactions between different forces when explaining the aggregate trends in the Brazilian labor market.

Validation: Empirical Spillovers. While spillovers along the formal wage distribution are well documented, there is less evidence on their effects along the informal wage distribution. Figure H.7 fills this gap and complements Figure 8: rather than focusing on a specific wage level, we compare wages conditional on quantiles of the baseline and counterfactual distributions, showing the percentage increase in wages at the q-th quantile of each sector. As noted by Haanwinckel (2025b), these graphs are not causal, since the same quantile may be occupied by

---

[34] We point to two main factors behind the preference to stay constrained by $w_{\min}$ in the formal sector. First, formal firms have an exit rate that is almost one third that of informal firms, which means that the gradient of $\Pi_1(\theta)$ is generally steeper than that of $\Pi_2(\theta)$. Second, this is compounded by the fact that we have entry costs, and so the formal threshold is shifted to the right – where the value function is even steeper due to the convexity of $\pi_1(p)$. Without entry costs and dynamic considerations, more firms – that prefer to be constrained by $w_{\min}$ in the formal sector in our estimation – may be attracted towards informality. Figure H.4 displays the differences in the net entry values of entry, highlighting how the derivative of their difference increases as we shift along the distribution of θ. We discuss this more in Appendix F.

[35] Messina and Silva (2017) estimate an improvement of 77% in terms of trade and 81% in domestic demand for the Brazilian economy in this period. Haanwinckel and Soares (2021) estimates a 24% larger productivity parameter, on top of a 12 p.p. increase in the fraction of skilled workers and 93% increase in a parameter that governs skill intensity in the production function. Ferreira and Veloso (2013) estimate a TFP increase of 22%.

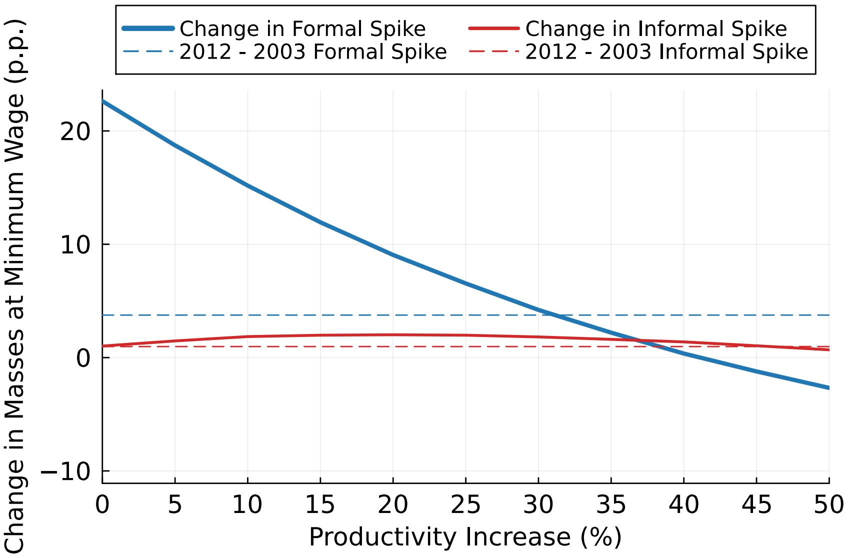


Figure 9: Minimum Wage Spikes and Productivity Increases

Notes: The figure shows the minimum wage spikes in the formal and informal sectors as a function of the percentage increase x in productivity ($(1+x)\theta$). Counterfactuals do not consider changes in V^U and fix the entry decisions of firms as in our main counterfactual with wage-anchoring. *Source*: model simulations and PNAD (2003, 2012).

different workers before and after the minimum wage increase.

We document relatively modest spillover effects in the formal sector, with wage increases decreasing until the 3rd decile and stabilizing thereafter.[36] Wage gains across the informal wage distribution are non-monotonic and heavily influenced by the existence of wage-anchoring. Figure H.7e compares the real wage gains between 2003 and 2012 in each quantile of the distribution in each sector. While obviously affected by different factors such as changes in productivity, skill-biased technological changes, formalization, and workforce composition, the empirical patterns are consistent with minimum wage spillovers being decreasing in the formal sector and non-monotonic in the informal sector.

7.1 Robustness

We assess the robustness of our results in two ways: by considering an alternative functional form for the labor supply curve, and by varying the width of the missing mass in the informal sector, following the results in Appendix B. For computational reasons, both exercises use half the number of simulated firms and datasets.

Labor Supply Curve. As noted in Section 6.1, there is surprisingly little work on the shape of labor supply curves, with the constant elasticity parametrization in Equation 15 being the most common in the literature mainly due to its simplicity. In this section, we consider an alternative

---

[36] These effects broadly agree with estimated spillovers in Haanwinckel (2025b, Figure 6) and are smaller than the ones in Engbom and Moser (2022, Figure 9) and Parente, Brotherhood, and Iachan (2025, Figure 12).

"logit" parametrization discussed in Kline (2025):

$$\ell_i(w) = \Lambda_i \left[1 + \exp(-\eta \log(w/\xi))\right]^{-1} = \Lambda_i \frac{w^\eta}{w^\eta + \xi^\eta}$$

where η and ξ are shape and scale parameters, respectively. Note that the wage elasticity approaches η as $w \to 0$ and 0 as $w \to \infty$, with ξ disciplining how fast the elasticity decreases.

As before, we restrict η and ξ to be the same across both sectors. To identify the extra parameter, we rely on the observation that a higher ξ makes elasticities flatter along wages w, decreasing firm sizes. Hence, we include moments from the CDF of firm sizes in the formal and informal sectors, collected from RAIS and ECINF, respectively. We also test estimating the model with the same moments as our main specifications.

Table 7 shows the main counterfactual results, while Tables H.2–H.4 show the estimates, model fit, and full counterfactuals. The model fit reveals that the specification with firm size moments can accurately capture the firm size distributions, although it underestimates the prevalence of larger firms in the formal sector. When using firm size moments, the effects on informality are larger because of the higher estimated η and ξ, while the predicted change in the formal minimum wage spike is smaller. Despite these varying magnitudes, the conclusions remain unchanged: the minimum wage causes a small increase in informality while reducing wage inequality in both sectors.

Width of Missing Mass in Informal Sector. The results in Appendix B show that our moments for the width of missing mass are imprecisely estimated, particularly in the informal sector. To account for this uncertainty, we conduct two robustness exercises. First, we decrease the width of missing mass in the informal sector from 105 to 75, which is the alternative mode in the bootstrap distribution in Figure B.2 and the value that minimizes the difference between excess and missing masses, rather than the first value at which $EM \leq MM$. Second, we decrease it further to 47, the estimate obtained from the alternative parametric lognormal procedure (Table B.5). Further details on the estimation of these objects are in Appendix B.

As expected, Table H.2 shows that decreasing the width moments implies a lower κ_2, which in turn requires a lower Type-A share σ_2 to match the informal spike. Table 7 shows that the small effects on informality and unemployment are robust across specifications, as are the effects on formal and total wage inequality. The results on informal wage inequality, however, are sensitive to the width of the missing mass region and merit further discussion.

In our main specification with $\Delta^* w_2 = 105$, we find that the minimum wage *reduces* inequality in the informal sector by 1.3 p.p. This result is overturned when lighthouse effects are not accounted for, with inequality instead increasing by 1.5 p.p. to 1.6 p.p. Table 7 shows that this result depends on a sufficiently large missing mass region: informal inequality exhibits small increases of 0.4 and 0.6 p.p. with the smaller widths of 75 and 47, respectively, as the narrower reach of the minimum wage limits its ability to compress the wage distribution. Nonetheless, even with smaller widths, accounting for lighthouse effects attenuates the increase in informal wage inequality relative to the counterfactual without the informal spike. Indeed, a similar decomposition to that in Table 5 shows that wage-anchoring still has an inequality-reducing effect in the informal sector, though not strong enough to offset the inequality-increasing impact from formal sector switchers.

Table 7: Effects of a 60% Increase in the Minimum Wage – Robustness Exercises

	Logit – With Firm Sizes			Logit – No Firm Sizes			Informal Width 75			Informal Width 47		
	Base	Δ	Δ MC	Base	Δ	Δ MC	Base	Δ	Δ MC	Base	Δ	Δ MC
Informality and Unemployment (%)												
Informality (Firms)	69.1	+6.9	+6.9	68.7	+3.8	+3.8	68.9	+5.0	+5.0	68.6	+5.7	+5.7
Informality (Workers)	28.0	+2.8	+2.7	28.7	+1.7	+1.7	29.7	+2.4	+2.4	29.5	+2.7	+2.6
Unemployment	25.1	+3.6	+3.0	25.1	+1.7	+2.0	25.1	+2.5	+2.4	25.1	+3.2	+2.4
Minimum Wage Spike (%)												
Formal	11.5	+17.0	+17.0	10.5	+22.5	+22.4	10.7	+23.1	+23.1	10.9	+21.8	+22.0
Informal	13.6	+0.8	+0.8	14.8	+1.7	+1.7	15.0	+0.6	+0.6	15.0	+0.9	+0.8
Total	12.1	+12.6	+12.6	11.7	+16.8	+16.8	11.9	+17.0	+17.0	12.1	+16.1	+16.2
Wage Inequality ($\text{Var}(\log(w))$)												
Formal	100.0	-29.8	-30.0	100.0	-28.3	-28.3	100.0	-21.7	-21.7	100.0	-25.0	-25.1
Informal	100.0	-2.1	-2.1	100.0	-1.7	-1.7	100.0	+0.4	+0.4	100.0	+0.6	+0.6
Total	100.0	-10.8	-10.8	100.0	-12.7	-12.7	100.0	-9.6	-9.6	100.0	-9.7	-9.7

Notes: The table shows the effect of an increase of 60% in the real minimum wage w_{min}. Changes are in percentage points. "MC" (market changes) considers changes in V^U discussed in Section D.4. Wage inequality measures are calculated based on the distribution of log wages and are normalized by their estimated baseline or 2003 values. 2012 wages deflated to R$ 2003 using the yearly consumer price index (IPCA). *Source*: model simulations and PNAD (2003, 2012).

8 Conclusion

This paper studies the lighthouse effect – the large share of informal workers earning exactly the minimum wage despite not being subject to labor regulations – and its implications for the effects of the minimum wage on informality, unemployment, and wage inequality in Brazil. We argue that firms in the formal and informal sectors offer the minimum wage because it serves as a cost-saving tool and as a benchmark for fair pay. We embed this idea in a dual-sector model that rationalizes the minimum wage spike and missing masses in both sectors. We estimate the model using Brazilian data and use it to assess the effects of the 60% increase in the real minimum wage observed between 2003 and 2012.

Our main findings can be summarized as follows. First, modeling lighthouse effects matters: ignoring the mass of informal workers at the minimum wage affects the estimated structural parameters, which in turn overstates the effect of minimum wage increases on informal wage inequality. Second, the minimum wage increase had small effects on informality (+2.6 p.p., 8.8% of the baseline) and caused unemployment to rise by 2.4 p.p. (9.6% of the baseline) when considering changes in market conditions. Third, the higher minimum wage accounts for nearly the entire 25.4% decline in formal wage inequality and 38% of the 27.5% fall in total wage inequality between 2003 and 2012. Our findings point to the minimum wage acting as a strong inequality-reducing tool – especially at the bottom of the distribution – with effects that reach even workers not subject to it.

These results come with important caveats that point to directions for future research. First, while our framework is well-suited to study wage inequality, the assumption of homogeneous workers makes it unable to address redistribution, that is, which workers benefit most from the higher minimum wage. Second, we are silent about the optimal level and adjustment mechanism of the policy. Third, we focus on steady-state analyses: understanding the transition dynamics following minimum wage changes and their impacts on productivity and innovation is an important avenue for future work, especially in the Brazilian setting where the policy involves annual real adjustments.

Finally, the use of the minimum wage as a benchmark for wage-setting may have implications for competition and market power that go beyond our model. By serving as a salient benchmark, the minimum wage can facilitate tacit wage coordination among employers, even without explicit collusion: if many firms anchor their wages to the minimum, wage competition is effectively suppressed. The same mechanism may operate on the worker side: if workers come to expect the minimum wage as the prevailing wage in their segment of the labor market, they may be less likely to search for new jobs or demand higher wages, effectively accepting monopsony rents they might otherwise resist. Investigating these channels is a promising direction for future research.

REFERENCES

Acemoglu, Daron, and William B Hawkins. 2014. “Search with multi-worker firms.” *Theoretical Economics* 9 (3): 583–628.

Adda, Jérôme, Christian Dustmann, and Katrien Stevens. 2017. “The career costs of children.” *Journal of Political Economy* 125 (2): 293–337.

Akerlof, George A, and Janet L Yellen. 1985. “A near-rational model of the business cycle, with wage and price inertia.” *Quarterly Journal of Economics* 100 (Supplement): 823–838.

Albrecht, James, and Bo Axell. 1984. “An equilibrium model of search unemployment.” *Journal of Political Economy* 92 (5): 824–840.

Albrecht, James, Carlos Carrillo-Tudela, and Susan Vroman. 2018. “On-the-job search with match-specific amenities.” *Economics Letters* 162:15–17.

Albrecht, James, and Susan Vroman. 1992. “Dual labor markets, efficiency wages, and search.” *Journal of Labor Economics* 10 (4): 438–461.

———. 1998. “Nash equilibrium efficiency wage distributions.” *International Economic Review,* 183–203.

Altonji, Joseph, and Lewis Segal. 1996. “Small-sample bias in GMM estimation of covariance structures.” *Journal of Business & Economic Statistics* 14 (3): 353–366.

Altonji, Joseph, Anthony Smith, and Ivan Vidangos. 2013. “Modeling earnings dynamics.” *Econometrica* 81 (4): 1395–1454.

Alvarez, Bruna. 2024. “Shifting Paradigms: The Consequences of Misclassifying Employees as Entrepreneurs.” Working Paper. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4598482.

Alvarez, Bruna, João Paulo Pessoa, and André Portela. 2023. *Firm Size Distribution and Informality Effects of a Revenue-Dependent Tax Policy.* Technical report. São Paulo School of Economics. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4554425.

Alvarez, Jorge, Felipe Benguria, Niklas Engbom, and Christian Moser. 2018. “Firms and the decline in earnings inequality in Brazil.” *American Economic Journal: Macroeconomics* 10 (1): 149–189.

Amodio, Francesco, Pamela Medina, and Monica Morlacco. 2024. *Labor market power, self-employment, and development.* Technical report. RF Berlin-CReAM Discussion Paper Series. Available at: https://www.econstor.eu/bitstream/10419/304440/1/1905681372.pdf.

Andrews, Isaiah, Matthew Gentzkow, and Jesse M Shapiro. 2017. “Measuring the sensitivity of parameter estimates to estimation moments.” *Quarterly Journal of Economics* 132 (4): 1553–1592.

Araujo, Luis, Vladimir Ponczek, and André Portela Souza. 2016. “Informality in an economy with active labour courts.” *Applied Economics* 48 (30): 2868–2882.

Azar, José, Steven Berry, and Ioana Marinescu. 2022. *Estimating labor market power.* Working Paper w30365. National Bureau of Economic Research. Available at: https://www.nber.org/system/files/working_papers/w30365/w30365.pdf.

Azar, José, Emiliano Huet-Vaughn, Ioana Marinescu, Bledi Taska, and Till Von Wachter. 2024. “Minimum wage employment effects and labour market concentration.” *Review of Economic Studies* 91 (4): 1843–1883.

Azar, José, and Ioana Marinescu. 2024. “Monopsony power in the labor market: From theory to policy.” *Annual Review of Economics* 16 (1): 491–518.

Berger, David, Kyle Herkenhoff, Andreas Kostøl, and Simon Mongey. 2023. *An Anatomy of Monopsony: Search Frictions, Amenities and Bargaining in Concentrated Markets.* Working Paper 31149. National Bureau of Economic Research. Available at: http://www.nber.org/papers/w31149.

Berger, David, Kyle Herkenhoff, and Simon Mongey. 2022. “Labor Market Power.” *American Economic Review* 112 (4): 1147–1193.

———. 2025. “Minimum Wages, Efficiency, and Welfare.” *Econometrica* 93 (1): 265–301.

Best, Michael, and Henrik Kleven. 2018. “Housing market responses to transaction taxes: Evidence from notches and stimulus in the UK.” *Review of Economic Studies* 85 (1): 157–193.

Bhaskar, Venkataraman, and Ted To. 2003. “Oligopsony and the distribution of wages.” *European Economic Review* 47 (2): 371–399.

Boeri, Tito, Pietro Garibaldi, and Marta Ribeiro. 2011. “The lighthouse effect and beyond.” *Review of Income and Wealth* 57:S54–S78.

Bontemps, Christian, Jean-Marc Robin, and Gerard J Van den Berg. 2000. “Equilibrium search with continuous productivity dispersion: Theory and nonparametric estimation.” *International Economic Review* 41 (2): 305–358.

Bosch, Mariano, Luca Flabbi, William Maloney, and Mauricio Tejada. 2025. “Sources and Implications of Labor Market Informality: The Case of Minimum Wage Policies.” Working Paper.

Bosch, Mariano, and Marco Manacorda. 2010. “Minimum wages and earnings inequality in urban Mexico.” *American Economic Journal: Applied Economics* 2 (4): 128–149.

Brochu, Pierre R, David A Green, Thomas Lemieux, and James H Townsend. 2025. *The Minimum Wage, Turnover, and the Shape of the Wage Distribution.* Working Paper 33479. National Bureau of Economic Research. Available at: http://www.nber.org/papers/w33479.

Broecke, Stijn, Alessia Forti, and Marieke Vandeweyer. 2017. “The effect of minimum wages on employment in emerging economies: a survey and meta-analysis.” *Oxford Development Studies* 45 (3): 366–391.

Brotherhood, Luiz, Daniel Da Mata, Cezar Santos, Nezih Guner, and Philipp Kircher. 2024. “Labor Market Regulation and Informality.” Working Paper. Available at: https://publications.iadb.org/en/labor-market-regulation-and-informality.

Bruins, Marianne, James Duffy, Michael Keane, and Anthony Smith. 2018. “Generalized indirect inference for discrete choice models.” *Journal of Econometrics* 205 (1): 177–203.

Bulow, Jeremy I, and Lawrence H Summers. 1986. “A theory of dual labor markets with application to industrial policy, discrimination, and Keynesian unemployment.” *Journal of Labor Economics* 4 (3): 376–414.

Burdett, Kenneth, and Dale T Mortensen. 1998. “Wage differentials, employer size, and unemployment.” *International Economic Review,* 257–273.

Cai, Jing, Sai Luo, and Shing-Yi Wang. 2025. *Money or Monitoring: Evidence on Improving Worker Effort.* Working Paper 33977. National Bureau of Economic Research. Available at: http://www.nber.org/papers/w33977.

Campos-Vázquez, Raymundo M., Robert Duval-Hernández, M. Ehekatzin García-Guzmán, and Laura Juárez. 2026. “Do Workers Value Formal Jobs? A Discrete Choice Experiment in Mexico.” Working Paper.

Card, David. 1992. “Using regional variation in wages to measure the effects of the federal minimum wage.” *Industrial and Labor Relations Review* 46 (1): 22–37. Available at: https://davidcard.berkeley.edu/papers/fed-min-wage-var.pdf.

Card, David, Ana Rute Cardoso, Joerg Heining, and Patrick Kline. 2018. “Firms and labor market inequality: Evidence and some theory.” *Journal of Labor Economics* 36 (S1): S13–S70.

Cardoso, Adalberto, and Telma Lage. 2005. “A inspeção do trabalho no Brasil.” *Dados* 48:451–489.

Carrillo-Tudela, Carlos. 2009. “An equilibrium search model when firms observe workers’ employment status.” *International Economic Review* 50 (2): 485–506.

Cengiz, Doruk, Arindrajit Dube, Attila Lindner, and Ben Zipperer. 2019. “The effect of minimum wages on low-wage jobs.” *Quarterly Journal of Economics* 134 (3): 1405–1454.

Chetty, Raj. 2012. “Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply.” *Econometrica* 80 (3): 969–1018.

Colombi, Roberto. 1990. “A New Model of Income Distribution: The Pareto-Lognormal Distribution.” In *Income and Wealth Distribution, Inequality and Poverty,* edited by Camilo Dagum and Michele Zenga, 18–32. Berlin: Springer-Verlag.

Corbi, Raphael B., Renata Narita, Rafael Ferreira, and Danilo Souza. 2022. "Labor Courts, Job Search and Employment: Evidence from a Labor Reform in Brazil." Working Paper. Available at: https://ssrn.com/abstract=4121304.

Corseuil, Carlos Henrique, Miguel Foguel, and Marcos Hecksher. 2015. "Efeitos dos pisos salariais estaduais sobre o mercado de trabalho: uma nova abordagem empírica." *Economia Aplicada* 19 (1): 131–169.

Davis, Donald R, and James Harrigan. 2011. "Good jobs, bad jobs, and trade liberalization." *Journal of International Economics* 84 (1): 26–36.

DellaVigna, Stefano, and Matthew Gentzkow. 2019. "Uniform pricing in US retail chains." *Quarterly Journal of Economics* 134 (4): 2011–2084.

Derenoncourt, Ellora, François Gérard, Lorenzo Lagos, and Claire Montialoux. 2025. "Minimum Wages and Informality." Working Paper. Available at: https://www.dropbox.com/scl/fi/wbo519f69evu8zbr7nd21/DGLM_2025b.pdf?rlkey=587cxa713cwrzmznb08jm57ah&e=1&dl=0.

Derenoncourt, Ellora, and David Weil. 2025. "Voluntary minimum wages: The local labor market effects of national retailer policies." *Quarterly Journal of Economics,* qjaf016.

Diamond, Peter A. 1971. "A model of Price Adjustment." *Journal of Economic Theory* 3 (2): 156–168.

Dube, Arindrajit, Laura Giuliano, and Jonathan Leonard. 2019. "Fairness and frictions: The impact of unequal raises on quit behavior." *American Economic Review* 109 (2): 620–663.

Dube, Arindrajit, Jeff Jacobs, Suresh Naidu, and Siddharth Suri. 2020. "Monopsony in online labor markets." *American Economic Review: Insights* 2 (1): 33–46.

Dube, Arindrajit, and Attila Lindner. 2024. *Minimum Wage in the 21st Century.* Working Paper 32878. National Bureau of Economic Research. Available at: https://www.nber.org/papers/w32878.

Dube, Arindrajit, Alan Manning, and Suresh Naidu. 2018. *Monopsony and Employer Mis-Optimization Explain Why Wages Bunch at Round Numbers.* Working Paper 24991. National Bureau of Economic Research. Available at: http://www.nber.org/papers/w24991.

———. 2025. "Monopsony and Employer Misoptimization Explain Why Wages Bunch at Round Numbers." *American Economic Review* 115 (8): 2689–2721.

Engbom, Niklas, Gustavo Gonzaga, Christian Moser, and Roberta Olivieri. 2022. "Earnings inequality and dynamics in the presence of informality: The case of Brazil." *Quantitative Economics* 13 (4): 1405–1446.

Engbom, Niklas, and Christian Moser. 2022. "Earnings Inequality and the Minimum Wage: Evidence from Brazil." *American Economic Review* 112 (12): 3803–3847.

Ferreira, Pedro, and Fernando Veloso. 2013. "O desenvolvimento econômico brasileiro no pós-guerra." In *Desenvolvimento econômico: uma perspectiva brasileira,* edited by Fernando Veloso, Pedro Ferreira, Fábio Giambiagi, and Samuel Pessôa. Campus.

Finamor, Lucas. 2025. "Labor Market Informality, Risk, and Insurance." Job Market Paper. Available at: https://lucasfinamor.github.io/files/Finamor_Lucas_JMP.pdf.

Firpo, Sergio, and Alysson Portella. 2025. "Declining Wage Inequality in Developing Countries: The Case of Brazil." *Journal of Economic Literature* 63 (3): 875–915.

Flinn, Christopher. 2006. "Minimum wage effects on labor market outcomes under search, matching, and endogenous contact rates." *Econometrica* 74 (4): 1013–1062.

Flinn, Christopher, and Joseph Mullins. 2021. "Firms' Choices of Wage-Setting Protocols in the Presence of Minimum Wages." Working Paper. Available at: https://events.bse.eu/live/files/4061-firmchoices-draft-oct-2021pdf.

Giupponi, Giulia, Robert Joyce, Attila Lindner, Tom Waters, Thomas Wernham, and Xiaowei Xu. 2024. "The employment and distributional impacts of nationwide minimum wage changes." *Journal of Labor Economics* 42 (S1): S293–S333.

Giupponi, Giulia, and Stephen Machin. 2022. "Company Wage Policy in a Low-Wage Labor Market." Working Paper. Available at: https://cep.lse.ac.uk/pubs/download/dp1869.pdf.

Gomes, Diego, Felipe Iachan, and Cezar Santos. 2020. "Labor earnings dynamics in a developing economy with a large informal sector." *Journal of Economic Dynamics and Control* 113:103854.

Guerreiro, Joao, Jonathon Hazell, Chen Lian, and Christina Patterson. 2024. *Why Do Workers Dislike Inflation? Wage Erosion and Conflict Costs.* Working Paper 32956. National Bureau of Economic Research. Available at: http://www.nber.org/papers/w32956.

Guo, Junjie. 2024. "Sequential job posting and equilibrium wage dispersion with bunching." *Labour Economics* 87:102489.

Haanwinckel, Daniel. 2025a. "Does regional variation in wage levels identify the effects of a national minimum wage?" Working Paper. Available at: https://haanwinckel.github.io/files/WageLevelsMinWage_Haanwinckel.pdf.

———. 2025b. "Supply, demand, institutions, and firms: A theory of labor market sorting and the wage distribution." Working Paper, Conditionally Accepted at the American Economic Review. Available at: https://haanwinckel.github.io/files/SDIF_condAcc.pdf.

Haanwinckel, Daniel, and Rodrigo R Soares. 2021. "Workforce composition, productivity, and labour regulations in a compensating differentials theory of informality." *Review of Economic Studies* 88 (6): 2970–3010.

Hall, Robert E, and Alan B Krueger. 2012. “Evidence on the incidence of wage posting, wage bargaining, and on-the-job search.” *American Economic Journal: Macroeconomics* 4 (4): 56–67.

Harasztosi, Peter, and Attila Lindner. 2019. “Who pays for the minimum wage?” *American Economic Review* 109 (8): 2693–2727.

Hazell, Jonathon, Christina Patterson, Heather Sarsons, and Bledi Taska. 2022. *National Wage Setting.* Working Paper 30623. National Bureau of Economic Research. Available at: http://www.nber.org/papers/w30623.

Hinojosa, Paul Garcia. 2019. “The Role of the Minimum Wage on the Declining Wage Inequality in Latin America: Evidence from Brazil.” *Review of Economic Analysis* 11 (1): 39–82.

Hjort, Jonas, Xuan Li, and Heather Sarsons. 2025. “Across-Country Wage Compression in Multinationals.” Working Paper. Available at: https://jhjort.github.io/MyWebsite/HLS_WageCompression_032925.pdf.

Hornstein, Andreas, Per Krusell, and Giovanni L Violante. 2011. “Frictional wage dispersion in search models: A quantitative assessment.” *American Economic Review* 101 (7): 2873–2898.

Hurst, Erik, Patrick J Kehoe, Elena Pastorino, and Thomas Winberry. 2022. *The Distributional Impact of the Minimum Wage in the Short and Long Run.* Working Paper 30294. National Bureau of Economic Research. Available at: http://www.nber.org/papers/w30294.

Hwang, Hae-shin, Dale T Mortensen, and W Robert Reed. 1998. “Hedonic wages and labor market search.” *Journal of Labor Economics* 16 (4): 815–847.

International Labour Organization. 2024. *Global Wage Report 2024-25.* Technical report. International Labour Organization. Available at: https://www.ilo.org/sites/default/files/2024-11/GWR-2024_Layout_E_RGB_Web.pdf.

Jales, Hugo. 2018. “Estimating the effects of the minimum wage in a developing country: A density discontinuity design approach.” *Journal of Applied Econometrics* 33 (1): 29–51.

Jales, Hugo, and Zhengfei Yu. 2024. “Minimum wage and Informality in a Roy Bargaining Economy: Evidence from a Bunching Estimator.” R&R at the Journal of Labor Economics. Available at: https://drive.google.com/file/d/1hg8Tl5CEVaEzmkWGGI_pmZEXip23Qvtl/view.

Kleven, Henrik. 2016. “Bunching.” *Annual Review of Economics* 8 (1): 435–464.

Kleven, Henrik, and Mazhar Waseem. 2013. “Using notches to uncover optimization frictions and structural elasticities: Theory and evidence from Pakistan.” *Quarterly Journal of Economics* 128 (2): 669–723.

Kline, Patrick. 2025. *Labor Market Monopsony: Fundamentals and Frontiers.* Discussion Paper 36/25. RFBerlin. Available at: https://eml.berkeley.edu/~pkline/papers/HoLE_monopsony.pdf.

Kopczuk, Wojciech, and David Munroe. 2015. "Mansion tax: The effect of transfer taxes on the residential real estate market." *American Economic Journal: Economic Policy* 7 (2): 214–257.

La Porta, Rafael, and Andrei Shleifer. 2014. "Informality and development." *Journal of Economic Perspectives* 28 (3): 109–126.

Lachowska, Marta, Alexandre Mas, Raffaele Saggio, and Stephen Woodbur. 2021. *Wage posting or wage bargaining? A test using dual jobholders.* Working Paper 28409. National Bureau of Economic Research. Available at: https://www.nber.org/system/files/working_papers/w28409/w28409.pdf.

Lamadon, Thibaut, Magne Mogstad, and Bradley Setzler. 2022. "Imperfect competition, compensating differentials, and rent sharing in the US labor market." *American Economic Review* 112 (1): 169–212.

Lemos, Sara. 2009. "Minimum wage effects in a developing country." *Labour Economics* 16 (2): 224–237.

Lombardo, Carlo, Lucía Ramírez Leira, and Leonardo Gasparini. 2024. "Does the minimum wage affect wage inequality? A study for the six largest Latin American economies." *Journal of Development Studies,* 1–17. Available at: https://www.cedlas.econo.unlp.edu.ar/wp/wp-content/uploads/doc_cedlas302.pdf.

Luduvice, André, Tomás Martinez, and Alexandre Sollaci. 2024. *Minimum Wage, Business Dynamism, and the Life Cycle of Firms.* Technical report. Inter-American Development Bank. Available at: https://publications.iadb.org/en/minimum-wage-business-dynamism-and-life-cycle-firms.

Maloney, William F., and Jorge Méndez. 2004. "Minimum Wages in Latin America." In *Law and Employment: Lessons from Latin America and the Caribbean,* edited by James J. Heckman and Carmen Pages, 231–259. Chicago: University of Chicago Press.

Martinez, Tomás, and Cezar Santos. 2025. *A Model of Market Power with Formal and Informal Firms.* Working Paper. Inter-American Development Bank. Available at: https://publications.iadb.org/en/model-market-power-formal-and-informal-firms-discussion-paper.

Meghir, Costas, Renata Narita, and Jean-Marc Robin. 2015. "Wages and informality in developing countries." *American Economic Review* 105 (4): 1509–1546.

Melitz, Marc J. 2003. "The impact of trade on intra-industry reallocations and aggregate industry productivity." *Econometrica* 71 (6): 1695–1725.

Messina, Julián, and Joana Silva. 2017. *Wage inequality in Latin America: Understanding the past to prepare for the future.* World Bank Publications.

Narita, Renata. 2020. "Self-employment in developing countries: A search-equilibrium approach." *Review of Economic Dynamics* 35:1–34.

Neri, Marcelo, Gustavo Gonzaga, and José Camargo. 2001. "Salário mínimo, efeito-farol e pobreza." *Brazilian Journal of Political Economy* 21:263–276.

Neumark, David, and Luis Felipe Munguia Corella. 2021. "Do minimum wages reduce employment in developing countries? A survey and exploration of conflicting evidence." *World Development* 137:105165.

Ortoleva, Pietro. 2013. "The price of flexibility: Towards a theory of thinking aversion." *Journal of Economic Theory* 148 (3): 903–934.

Parente, Rafael, Lucas Brotherhood, and Felipe Iachan. 2025. "Minimum Wages, Inequality and the Informal Sector." Working Paper. Available at: https://parenterafael.github.io/papers/Minw_Ineq_Inf.pdf.

Pérez, Jorge Pérez. 2020. "The minimum wage in formal and informal sectors: Evidence from an inflation shock." *World Development* 133:104999.

Piyapromdee, Suphanit. 2018. "Residual wage dispersion with efficiency wages." *International Economic Review* 59 (3): 1315–1343.

Postel–Vinay, Fabien, and Jean–Marc Robin. 2002. "Equilibrium wage dispersion with worker and employer heterogeneity." *Econometrica* 70 (6): 2295–2350.

Ramaswamy, Ramana, and Robert E Rowthorn. 1991. "Efficiency wages and wage dispersion." *Economica,* 501–514.

Rauch, James E. 1991. "Modelling the informal sector formally." *Journal of Development Economics* 35 (1): 33–47.

Rebitzer, James. 1995. "Is there a trade-off between supervision and wages? An empirical test of efficiency wage theory." *Journal of Economic Behavior & Organization* 28 (1): 107–129.

Rebitzer, James, and Lowell Taylor. 1995. "The consequences of minimum wage laws: some new theoretical ideas." *Journal of Public Economics* 56 (2): 245–255.

Reyes, Germán. 2024. "Coarse wage-setting and behavioral firms." *Review of Economics and Statistics,* 1–41.

Rogerson, Richard, Robert Shimer, and Randall Wright. 2005. "Search-Theoretic Models of the Labor Market: A Survey." *Journal of Economic Literature* 43 (4): 959–988.

Saltiel, Fernando, and Sergio Urzúa. 2022. "Does an increasing minimum wage reduce formal sector employment? Evidence from Brazil." *Economic Development and Cultural Change* 70 (4): 1403–1437.

Schymura, Luiz Guilherme. 2019. "Implicações da regra de reajuste do salário mínimo." *Blog do Instituto Brasileiro de Economia,* Available at: https://blogdoibre.fgv.br/posts/implicacoes-da-regra-de-reajuste-do-salario-minimo.

Shapiro, Carl, and Joseph E Stiglitz. 1984. "Equilibrium unemployment as a worker discipline device." *American Economic Review* 74 (3): 433–444.

Shorrocks, Anthony. 2013. "Decomposition procedures for distributional analysis: a unified framework based on the Shapley value." *Journal of Economic Inequality* 11 (1): 99–126.

Sokolova, Anna, and Todd Sorensen. 2021. "Monopsony in Labor Markets: a meta-analysis." *ILR Review* 74 (1): 27–55.

Ulyssea, Gabriel. 2010. "Regulation of entry, labor market institutions and the informal sector." *Journal of Development Economics* 91 (1): 87–99.

———. 2018. "Firms, Informality, and Development: Theory and Evidence from Brazil." *American Economic Review* 108 (8): 2015–2047.

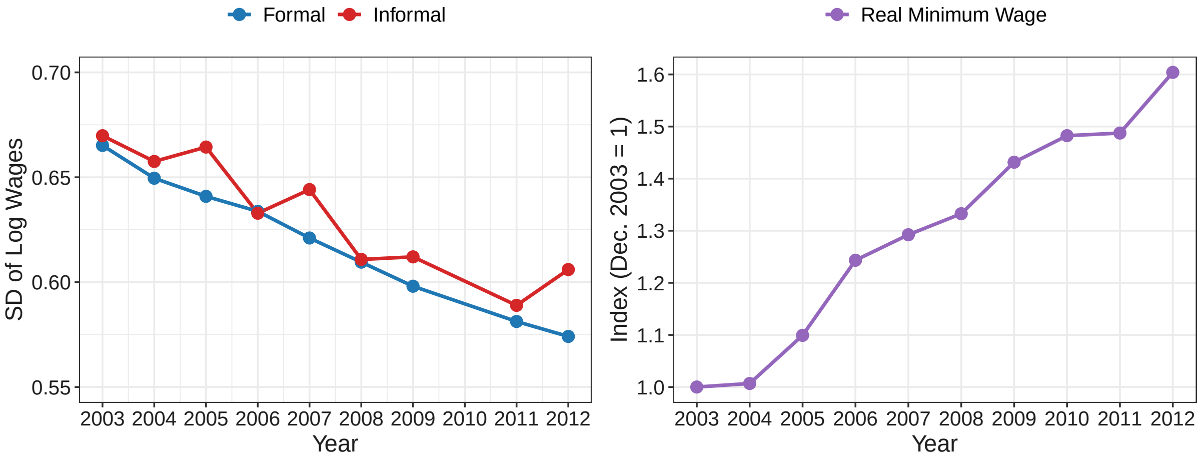
———. 2020. "Informality: Causes and consequences for development." *Annual Review of Economics* 12 (1): 525–546.

Van den Berg, Gerard J. 2003. "Multiple equilibria and minimum wages in labor markets with informational frictions and heterogeneous production technologies." *International Economic Review* 44 (4): 1337–1357.

Van den Berg, Gerard J, and Geert Ridder. 1998. "An empirical equilibrium search model of the labor market." *Econometrica,* 1183–1221.

Appendices

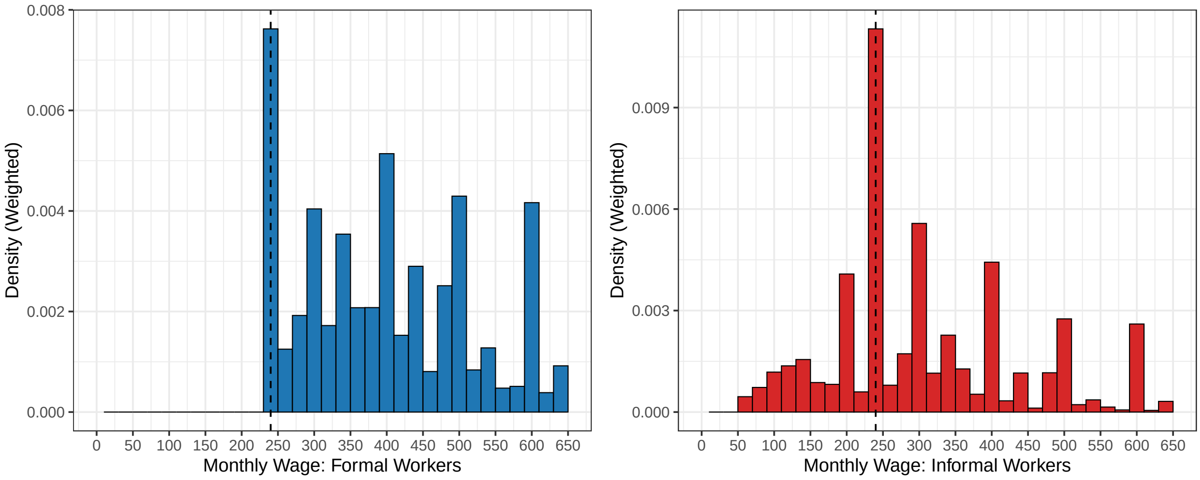
A Additional Figures and Tables



(a) Wage Dispersion (b) Real Minimum Wage

Figure A.1: Wage Inequality and Minimum Wage Trends in Brazil (2003–2012)

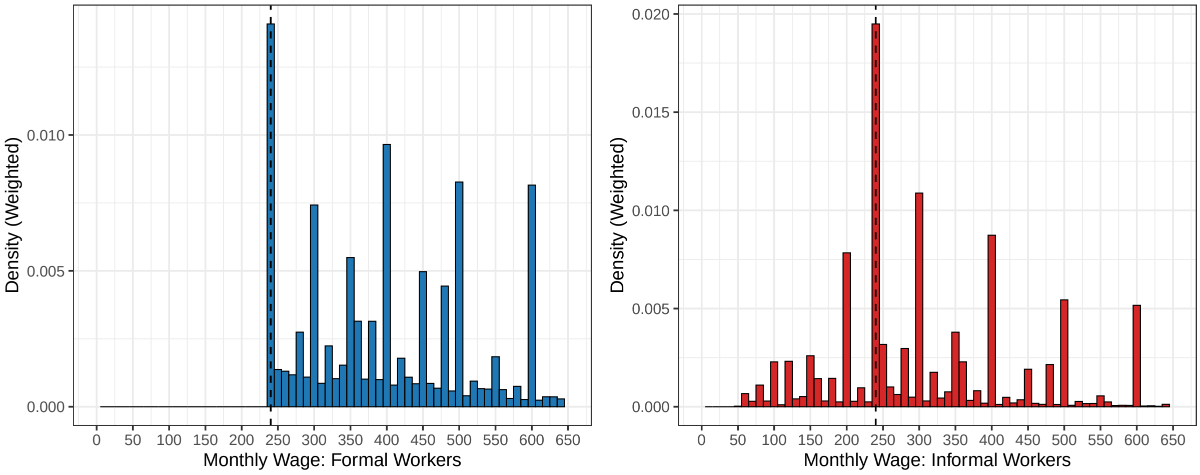
Notes: Wages are deflated to R$ 2003 using the yearly consumer price index (*Índice de Preços ao Consumidor Amplo*, IPCA). "SD": standard deviation. Minimum wages calculated in December of each year. Sample selection as described in Section 2.
Sources: PNAD (2003–2012) and Ipeadata (nominal minimum wage and consumer price index).



(a) Formal Workers (b) Informal Workers

Figure A.2: Formal and Informal Wage Histograms – PNAD 2003 (Binwidth: R$ 20)

Notes: The figure shows the wage histograms in PNAD using a binwidth of R$ 20 and truncated at R$ 650. Sample selection as described in Section 2.
Source: PNAD (2003). Use this to return to Section 2.

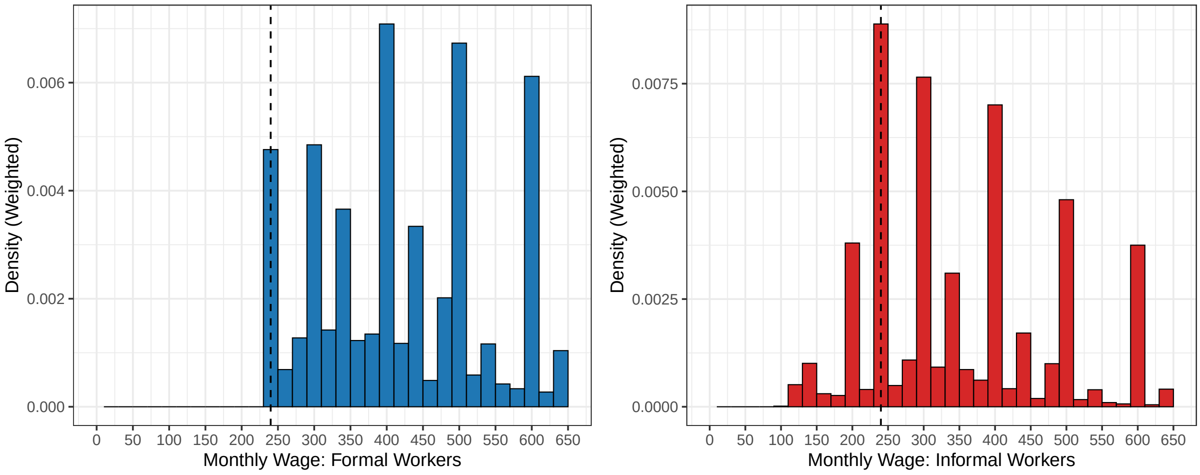


(a) Formal Workers (b) Informal Workers

Figure A.3: Formal and Informal Wage Histograms – PNAD 2003 (Binwidth: R$ 10)

Notes: The figure shows the wage histograms in PNAD using a binwidth of R$ 10 and truncated at R$ 650. Sample selection as described in Section 2.

Source: PNAD (2003). Use this to return to Section 2.

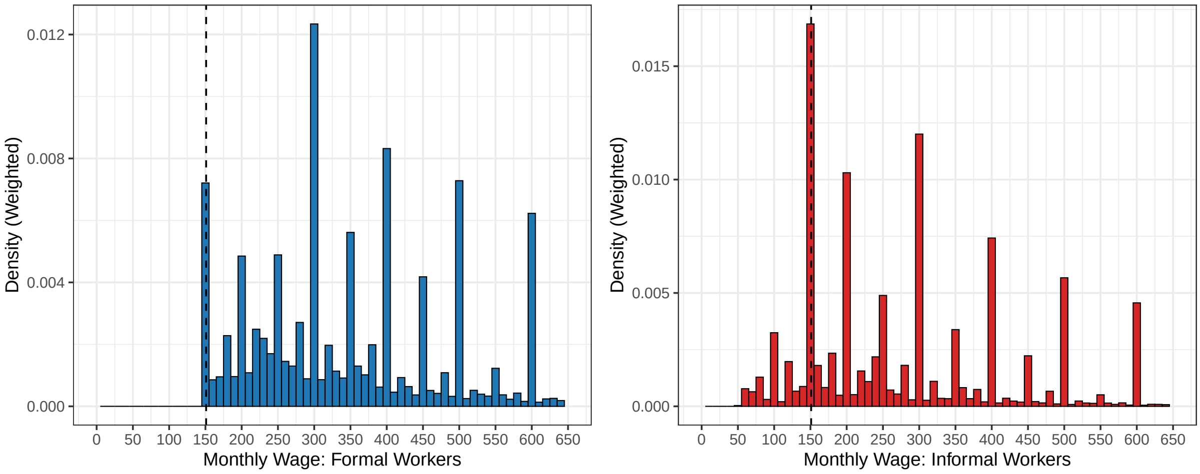


(a) Formal Workers (b) Informal Workers

Figure A.4: Formal and Informal Wage Histograms – PME

Notes: The figure shows the wage histograms in PME using a binwidth of R$ 20 and truncated at R$ 650. Sample selection as described in Section 2.

Source: PME (2003). Use this to return to Section 2.



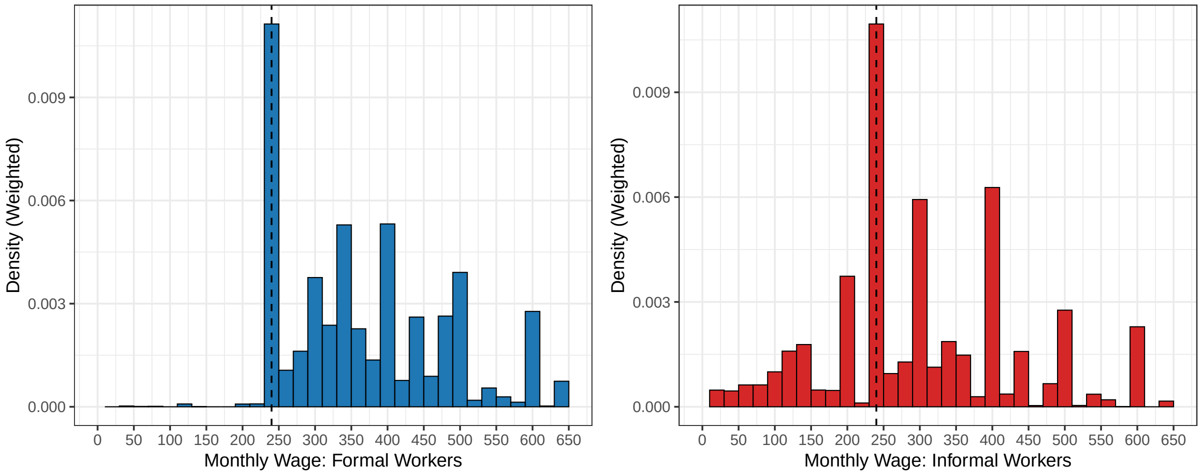
(a) Formal Workers

(b) Informal Workers

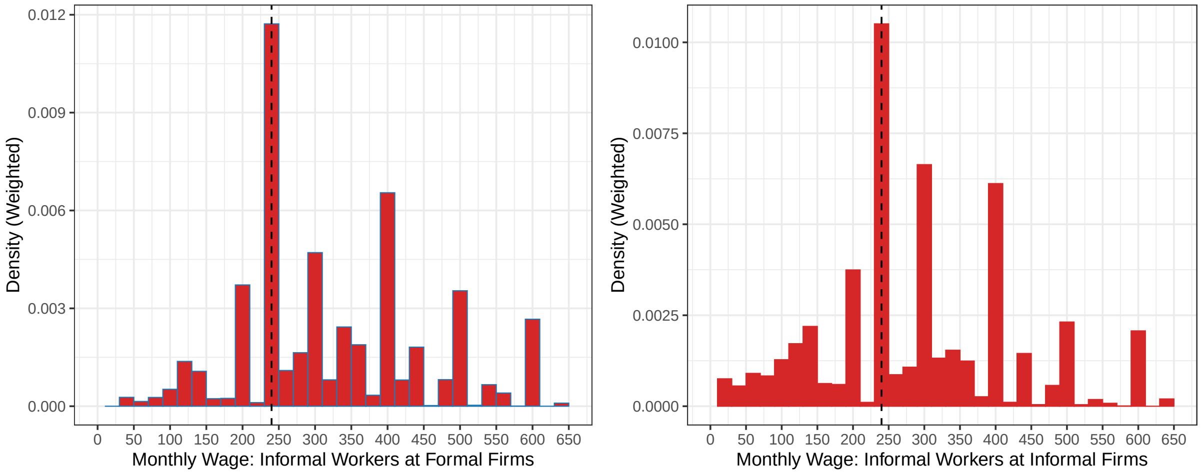
Figure A.5: Formal and Informal Wage Histograms – 2000 Census (MW: R$ 151)

Notes: The figure shows the wage histograms in the 2000 Census using a binwidth of R$ 10 and truncated at R$ 650. Sample selection as described in Section 2.

Source: Census (2000). Use this to return to Section 2.



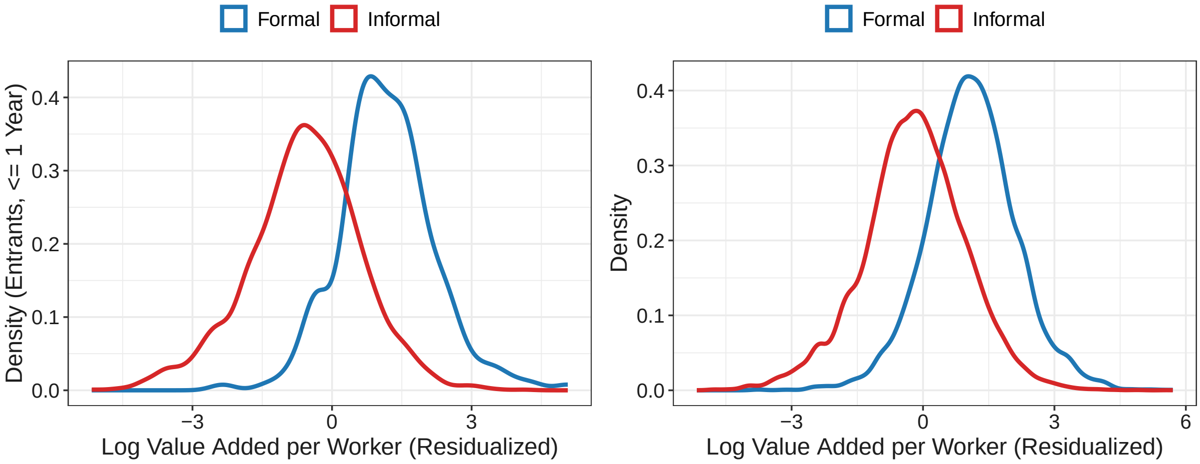
(a) Formal Workers – All Firms (b) Informal Workers – All Firms



(c) Informal Workers – Formal Firms (d) Informal Workers – Informal Firms

Figure A.6: Formal and Informal Wage Histograms – ECINF

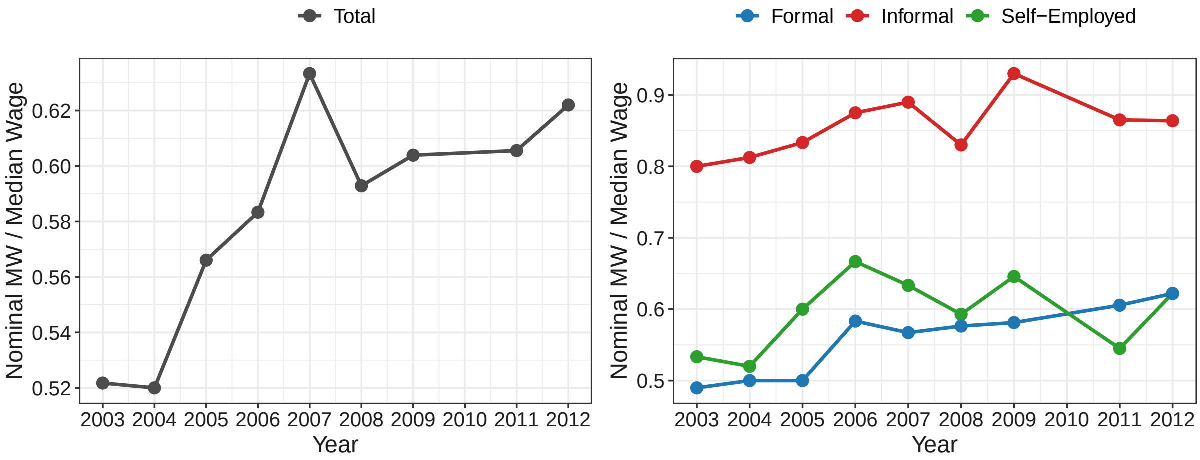
Notes: The figure shows the wage histograms in ECINF using a binwidth of R$ 20 and truncated at R$ 650. Distributions conditional on being a formal or informal worker (excluding owners and self-employed) and on having a wage greater than R$ 10. Worker status is expressed as the fill of bars, with firm status as the outline. Sample selection as described in Section 2.
Source: ECINF (2003). Use this to return to Section 2.



(a) Entrants (b) All Firms

Figure A.7: Distribution of Firm's Value Added per Worker (Proxy for Productivity)

Notes: The graphs follow Ulyssea (2018) and show the distribution of the log of value added per worker for entrants – firms with less than one year of activity – and all firms in each sector. Value added per worker proxies for productivity and is calculated as total revenue minus the cost of materials divided by the number of people in the firm. Distributions are conditional on having a positive value added measure and are residualized from industry dummies (manufacturing, retail and services). Sample selection as described in Section 2; importantly, ECINF is only representative of firms with up to five workers, although it contains information on firms with up to ten; in the figure, we consider firms with up to seven people, as in Ulyssea (2018).
Source: ECINF (2003). Use this to return to Section 2.



(a) Total (b) By Sector

Figure A.8: Minimum Wage Bite: Ratio with Median Wage

Notes: The figure shows the ratio between the minimum wage and the median wage across years. Wages are deflated to R$ 2003 using the consumer price index. "Total" includes formal and informal workers, self-employed, and employers. Sample selection as described in Section 2.
Source: PNAD. Use this to return to Section 2.

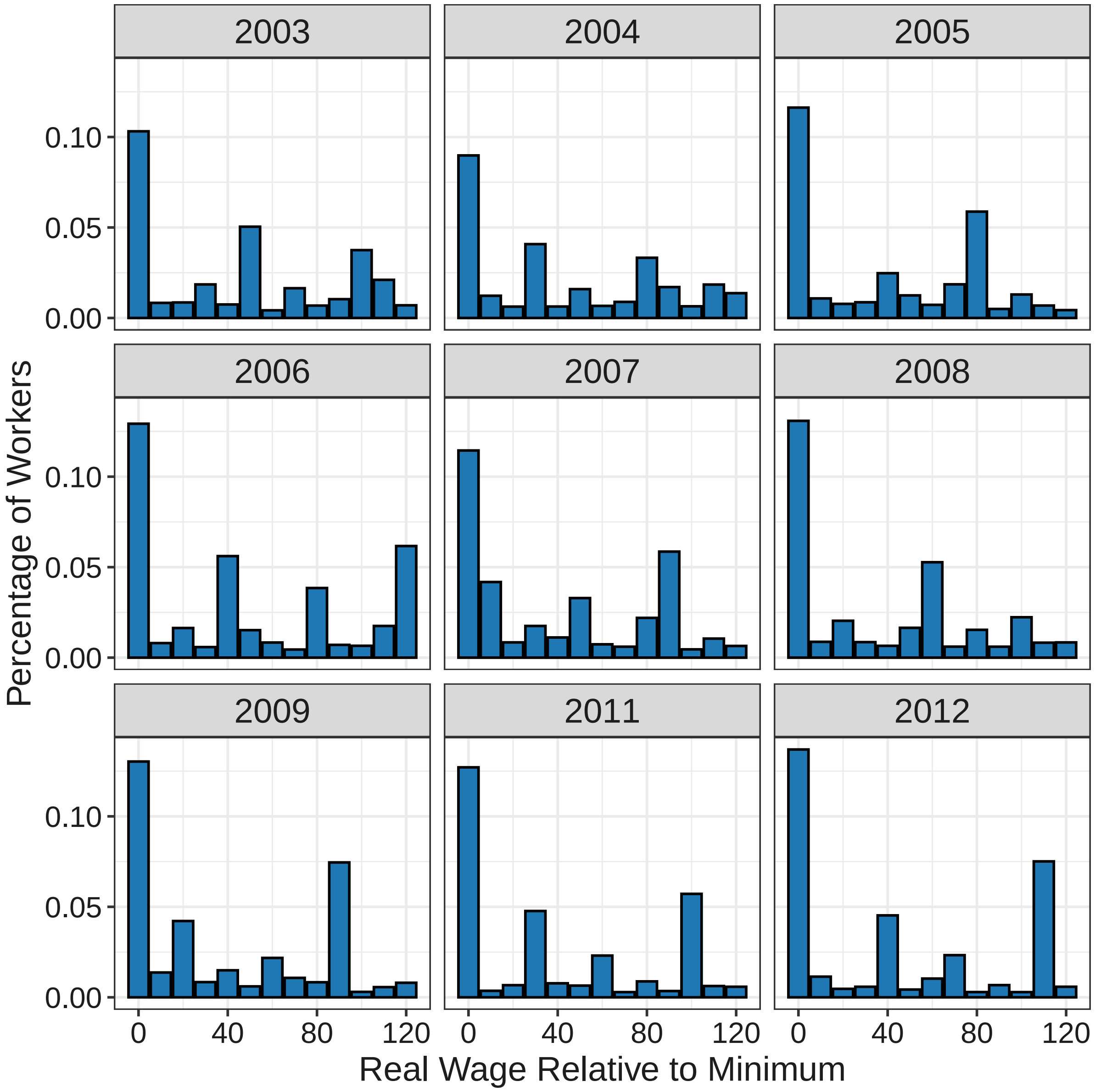


Figure A.9: Formal Minimum Wage Spike by Year: PNAD (2003–2012)

Notes: The figure shows the histograms of the wage distributions of each year 2003–2012, expressing them as deviations from each year's minimum wage, and deflating them to 2003 values using the Consumer Price Index (IPCA). PNAD uses a binwidth of R$ 10. Sample selection as described in Section 2.

Sources: PNAD (2003–2012). Use this to return to Section 3.

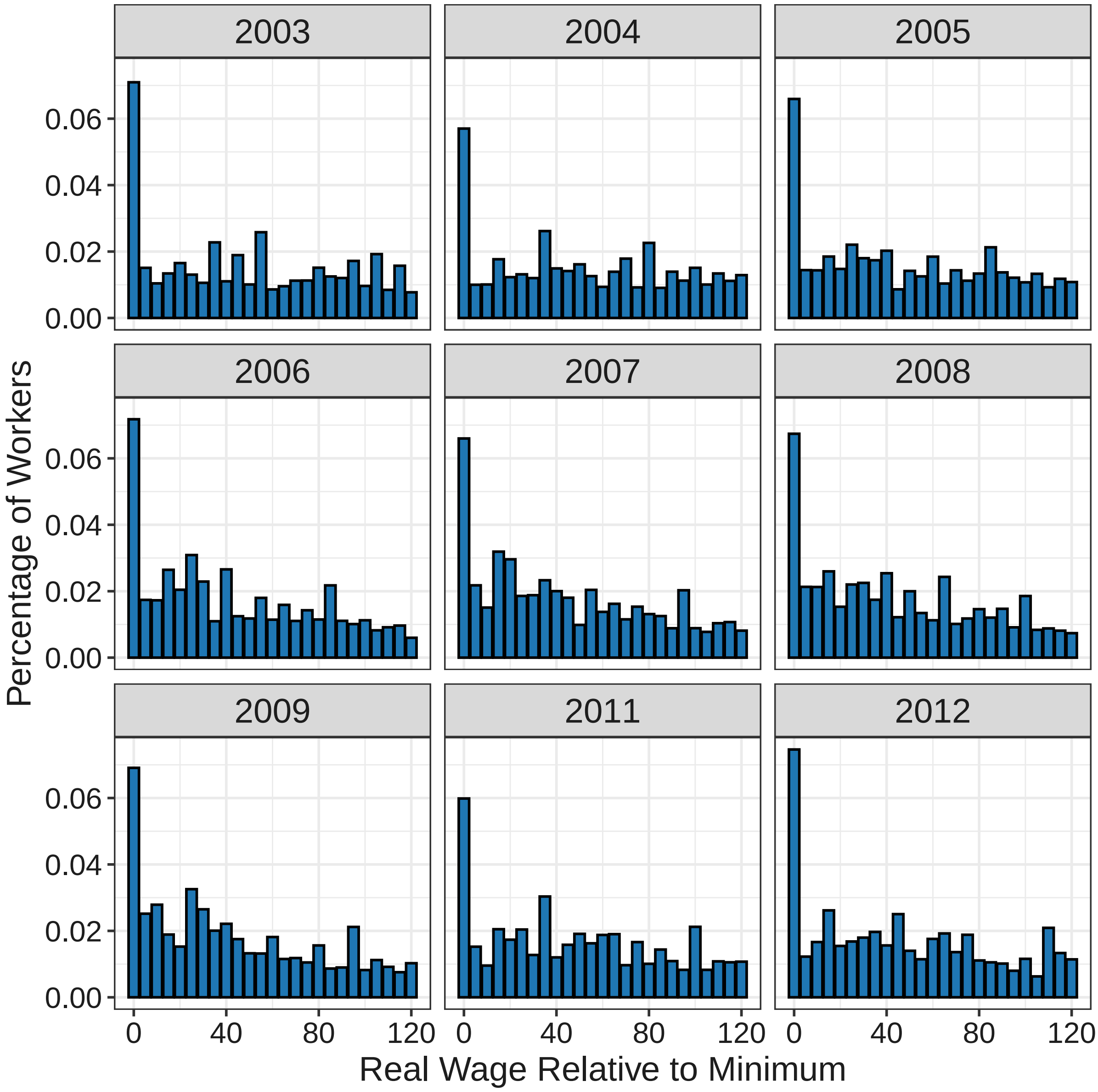


Figure A.10: Formal Minimum Wage Spike by Year: RAIS (2003–2012)

Notes: The figure shows the histograms of the wage distributions of each year 2003–2012, expressing them as deviations from each year's minimum wage, and deflating them to 2003 values using the Consumer Price Index (IPCA). RAIS uses a binwidth of R$ 5. Sample selection as described in Section 2.

Sources: RAIS (2003–2012). Use this to return to Section 3.

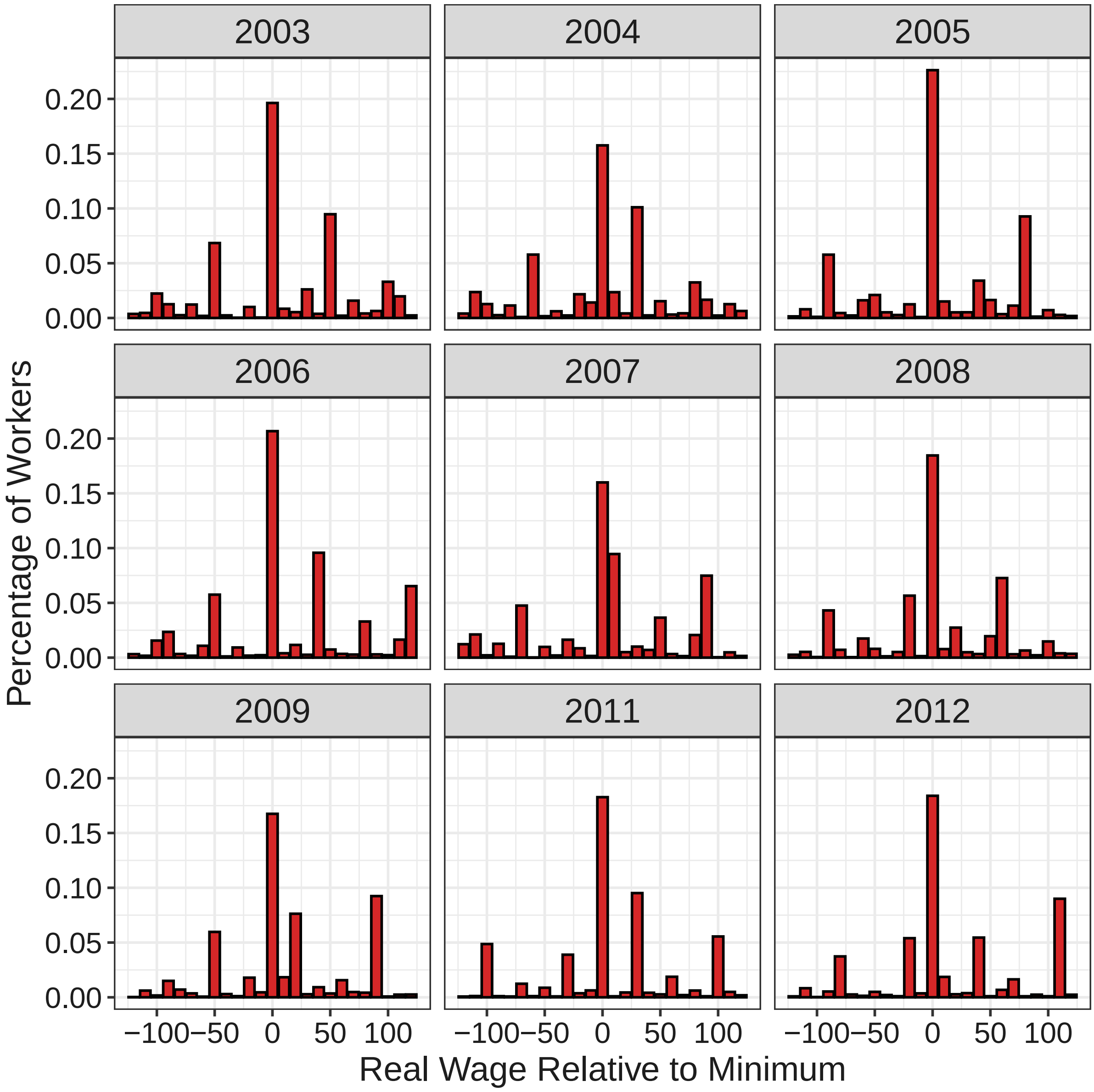
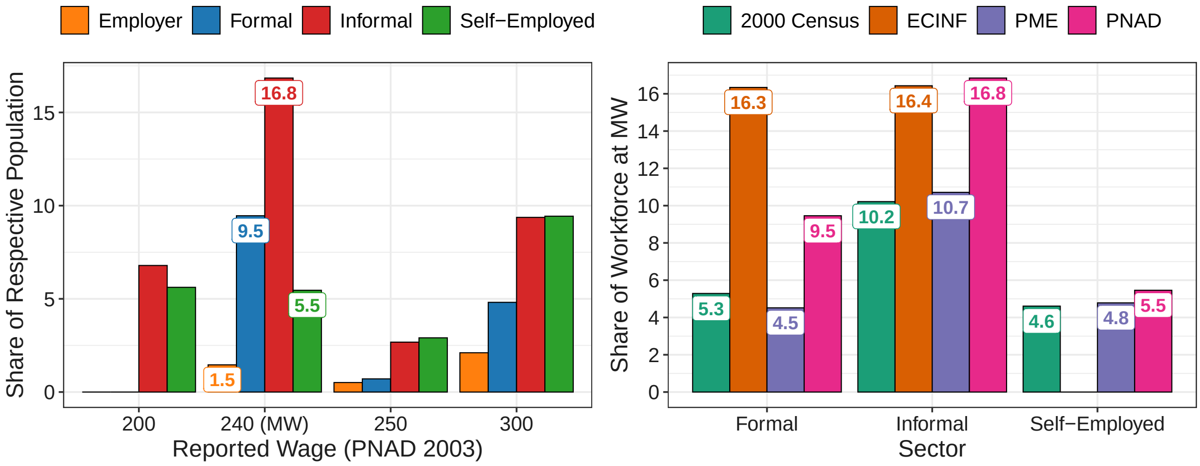


Figure A.11: Informal Minimum Wage Spike by Year: PNAD (2003–2012)

Notes: The figure shows the histograms of the wage distributions of each year 2003–2012, expressing them as deviations from each year's minimum wage, and deflating them to 2003 values using the Consumer Price Index (IPCA). PNAD uses a binwidth of R$ 10. Sample selection as described in Section 2.

Sources: PNAD (2003–2012). Use this to return to Section 3.

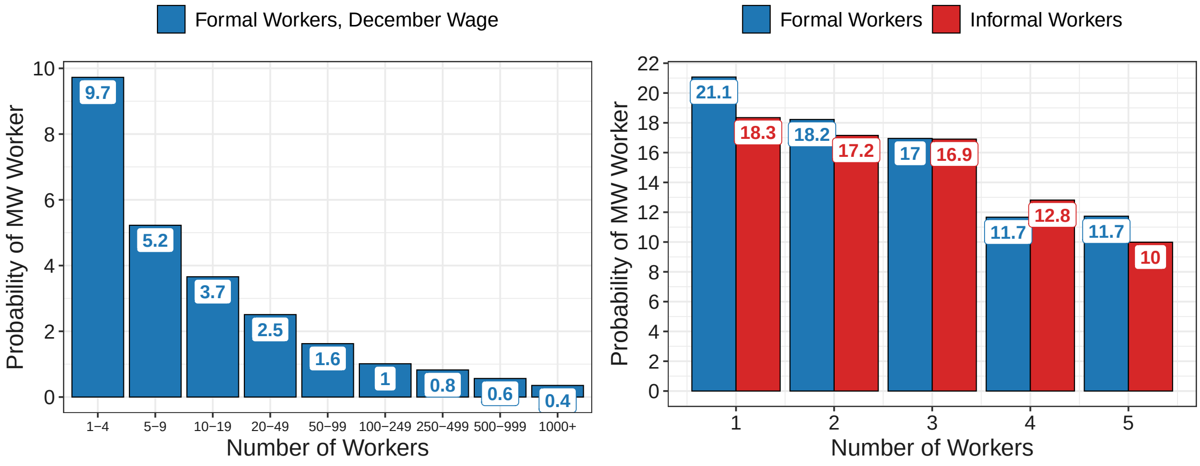


(a) Minimum Wage Spike and Close Round Numbers (b) Minimum Wage Spike Across Different Sources

Figure A.12: Minimum Wage Spikes: Closest Round Numbers and Different Sources (2003)

Notes: Calculations include workers who report earning exactly the nominal gross minimum wage (R$ 240 in 2003). Sample selection as described in Section 2.

Source: PNAD (2003), Census (2000), PME (2003), and ECINF (2003). Use this to return to Section 3.

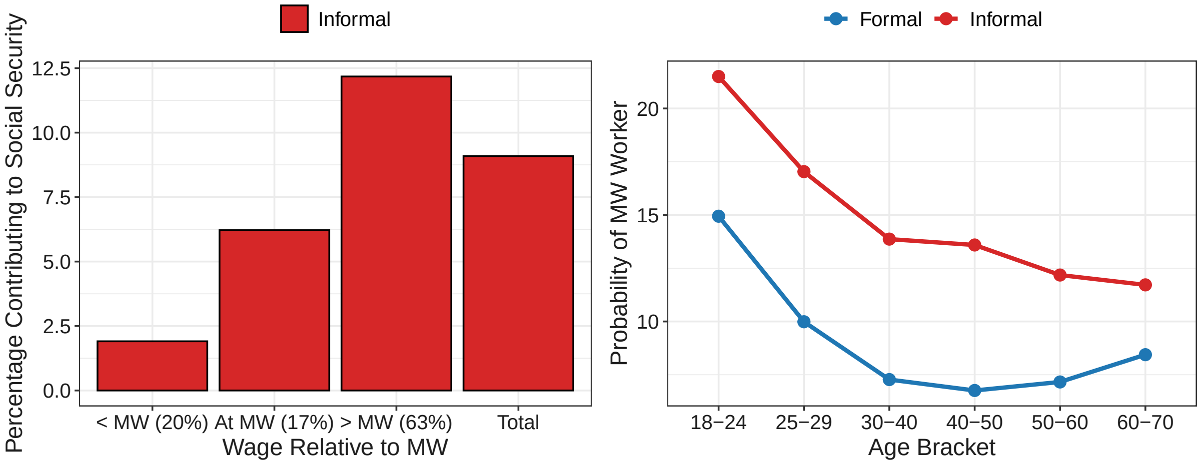


(a) RAIS – December Wages (b) ECINF

Figure A.13: Minimum Wage Spike by Firm Size

Notes: The figure shows the share of workers in each sector who report earning exactly the minimum wage as a function of firm size. December wages underestimate the formal spike (Derenoncourt et al. 2025, Appendix A4). Sample selection as described in Section 2.

Sources: RAIS and ECINF (2003). Use this to return to Section 4.1.

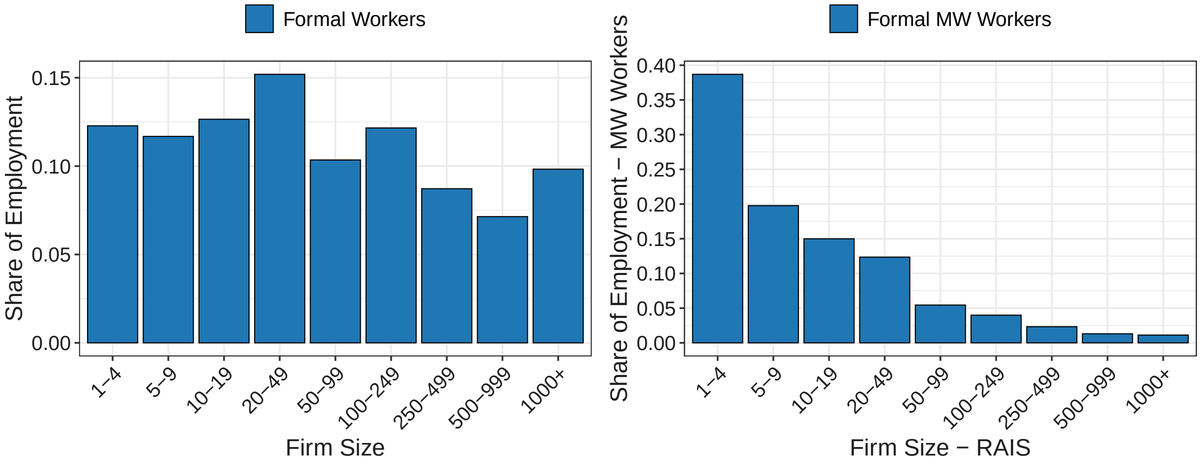


(a) Informal Workers Contributing to Social Security (b) Minimum Wage Spike by Age

Figure A.14: Social Security Contributions of Informal Workers and Minimum Wage Spike by Age

Notes: The left graph shows the share of informal workers that voluntarily contribute to social security, with the numbers on the x-axis denoting the share of the informal population in that category. All formal workers must contribute to social security. The right graph shows the share of workers in each sector earning exactly the minimum wage by age bracket. Sample selection as described in Section 2.

Source: PNAD (2003). Use this to return to Section 4.1.



(a) All Workers (b) Minimum Wage Workers

Figure A.15: Share of Total Formal Employment by Firm Size

Notes: The figure shows the share of formal employment by firm size. The left panel shows the distribution considering all workers, while the right figure focus on minimum wage workers. December wages underestimate the formal spike (Derenoncourt et al. 2025, Appendix A4). Sample selection as described in Section 2.

Source: RAIS (2003). Use this to return to Section D.2.3.

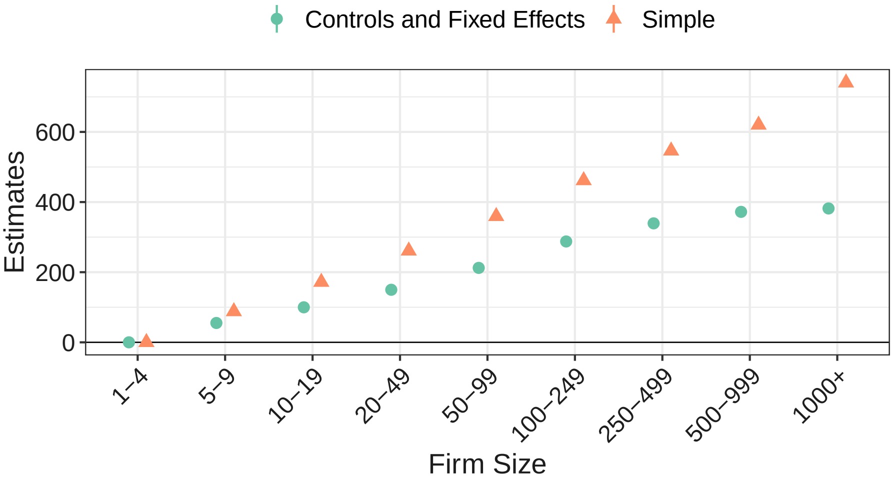


Figure A.16: Correlation Between Firm Sizes and Wages in the Formal Sector

Notes: The figure shows the firm size coefficients of a regression of worker wages on the number of employees of the firm they work for. "Simple" are the coefficients of a bivariate regression, while "Controls and Fixed Effects" include controls for tenure, contracted hours, age and fixed effects for state, schooling level, sex and activity (5-digit CNAEs). Sample selection as described in Section 2.
Source: RAIS (2003). Use this to return to Section D.2.3.

B Bunching Analysis

This appendix describes the bunching estimation used to verify the magnitude of the excess mass at the minimum wage in both sectors and to construct the width of missing mass moments used in our SMM estimation. We focus on the 2003 distribution to inform our SMM estimation and to account for potential reporting errors at round numbers ("heaping"); the results are analogous for the pooled 2003–2012 distributions discussed in Section 3.

Empirical Strategy. We make the standard assumptions that the partial counterfactual distribution is smooth and that the excess mass at the point of interest – in our case, the minimum wage – comes from the surrounding area in the distribution. In particular, we assume that the latent distribution is a $K = 6$th order polynomial, following Dube, Manning, and Naidu (2025). We then fit this polynomial to the observed wage distributions in PNAD, excluding a range around $w_{\min}$, and extrapolate the fitted values to the minimum wage.

Formally, we group individuals into wage bins of binwidth $b = 5$, separating by sector (formal or informal).[37] Let p_w be the share of workers in each sector with wage between w and $w+b$, that is, with wage in $[w, w+b)$. We estimate the following regression with OLS:

$$p_w = \sum_{j=w_{\min}-\Delta w}^{w_{\min}+\Delta w} \beta_j \mathbb{1}\{w = j\} + \sum_{k=0}^{K} \alpha_k w^k + \delta_w \mathbb{1}\{50 \mid w\} + \varepsilon_w, \tag{B.1}$$

where $\sum_{k=0}^{K} \alpha_k w^k$ is the flexible polynomial used to approximate the counterfactual distribution and Δw is the width of the area whose mass is transferred to the bunching – the *width of the missing mass*. β_j are the coefficients for the dummies in the excluded region: $\beta_{w_{\min}}$ is the excess mass (*EM*) at $w_{\min}$, $\sum_{j=w_{\min}-\Delta w}^{w_{\min}-b} \beta_j$ is the missing mass below $w_{\min}$ (*MMB*), and $\sum_{j=w_{\min}+b}^{w_{\min}+\Delta w} \beta_j$ is the missing mass above $w_{\min}$ (*MMA*). Finally, δ_w are round number fixed effects to flexibly control for heaping at bins that start at wages multiples of 50.[38] Note that, unlike Equation 1, Equation B.1 works with wages in levels (not in real terms relative to the minimum wage), as we are dealing with only one year (2003).

To be consistent with our data, we assume full compliance with the minimum wage in the formal sector, such that $MMB = 0$ and the first term becomes $\sum_{j=0}^{\Delta w} \beta_j \mathbb{1}\{w = j\}$. Therefore, the excess mass at the minimum wage beyond that predicted by the polynomial comes only from above $w_{\min}$. Implicitly, this assumes that the polynomial already accounts for workers who would have earned below $w_{\min}$ absent its bite, that is, for cases where the spike may come from censoring below $w_{\min}$. Note, however, that we *do not* assume that the entirety of the formal spike comes from the missing mass above the minimum wage. As Kleven and Waseem (2013) and Kleven (2016) argue, this bunching approach estimates a "partial counterfactual" stripped only

---

[37] We conduct robustness tests with respect to b and K at the end of this appendix.

[38] There is a perfect multicollinearity problem for wage bins that start at multiples of 50 in the excluded area. In these cases, β_j is not estimated and does not contribute to the missing mass.

of intensive margin responses, and not the full counterfactual absent $w_{\min}$.[39]

This would be a problem if we were interested in estimating structural elasticities, which is not the case. Furthermore, this partial counterfactual is the empirically equivalent object to the distribution of our model without wage-anchoring and is consistent with our goal of informing our SMM estimation of the width of the missing mass Δw above $w_{\min}$, which maps directly into the wage-anchoring cost κ_i. In our model, the "entry" channel for the spike already accounts for the excess mass that comes from formal firms wanting to pay a wage lower than $w_{\min}$, but not being able to; the "wage-anchoring" mechanism is responsible for the existence of the missing mass. In the informal sector, we implicitly assume symmetry of the excluded range Δw above and below $w_{\min}$, but not of the mass itself, as β_j are free to vary.

Width Estimation. Note that Δw is unknown. To determine it, we follow Kleven (2016) and use an iterative procedure. We start with $\Delta w = b$ and estimate EM and $MM = MMA + MMB$. If $EM > MM$, we update $\Delta w = 2b$ and redo the process. We continue increasing Δw by b until $EM \leq MM$, so that the missing mass equals (at least) the excess mass.

Let $\Delta^* w$ be the estimated width of the missing mass. To calculate its standard error, we follow Kleven (2016) and use a residual bootstrap: with $\Delta^* w$, we estimate Equation B.1 and store the predicted values $\hat{p}_w$ and the residuals $\hat{\varepsilon}_w$.[40] In each bootstrap replication r, we sample with replacement from $\hat{\varepsilon}_w$ to construct a vector of bootstrap residuals $\hat{\varepsilon}^r_w$. We use these residuals to form our bootstrap outcome $p^r_w = \hat{p}_w + \hat{\varepsilon}^r_w$ and redo the iterative procedure to get a new estimate $\Delta^{*r} w$. Our standard error for $\Delta^* w$ is the standard deviation of the bootstrap distribution $\{\Delta^{*r} w\}_{r=1}^{R}$ with $R = 1{,}000$ bootstrap replications.

Local Stability. Bunching is fundamentally a local approach: Equation B.1 is estimated using only an interval I around $w_{\min}$ – or above it in the formal sector. To determine this interval, we use a "local stability" approach, using the iterative procedure to estimate Equation B.1, restricting the data to different intervals until we reach a region where the estimated $\Delta^* w$ is invariant to changes in I.[41],[42] Note that, for smaller intervals, the algorithm need not converge.

The results are shown in Figure B.1. Reassuringly, the estimated $\Delta^* w$ in PNAD are constant in both sectors within $I \in [360, 440]$. The estimated $\Delta^* w$ for the formal sector using contractual wages in RAIS is remarkably close to the one in PNAD in this interval. For this reason, we opt for $I = 400$, which corresponds to 80 bins of $b = 5$. We keep I fixed in the bootstrap replications.

---

[39] Kopczuk and Munroe (2015) and Best and Kleven (2018) provide solutions to account for extensive margin responses, but their methods require data to the left and right of the notch, which we do not have in the formal sector.

[40] Note that $\hat{p}_w$ in this case is not the counterfactual distribution approximated by the polynomial $\sum_{k=0}^{K} \alpha_k w^k$, as it features both the dummies in the excluded region and the round number fixed effects. Furthermore, the bootstrap standard error is conditional on the choices of K, b, and I.

[41] We thank Enlinson Mattos at FGV–EESP for this suggestion.

[42] We calculate p_w before restricting the data, so it does not respond to changes in I. Additionally, we do not impute zeros if I stretches beyond the support of the data.

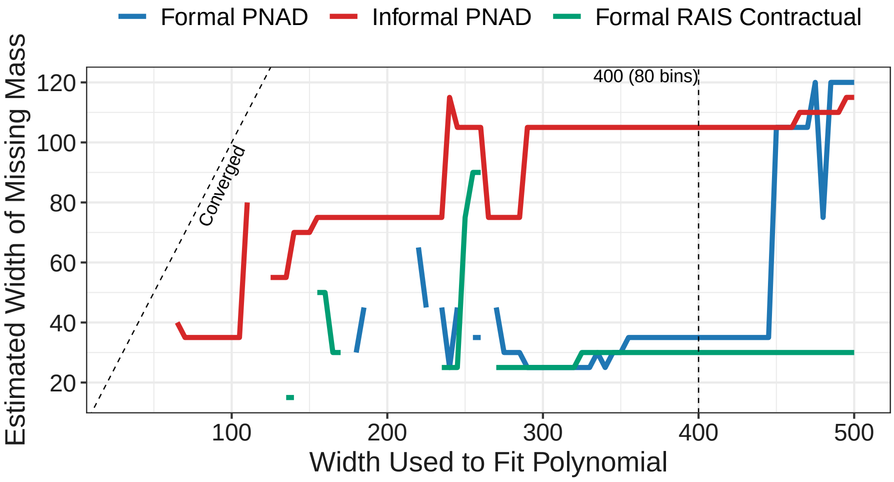
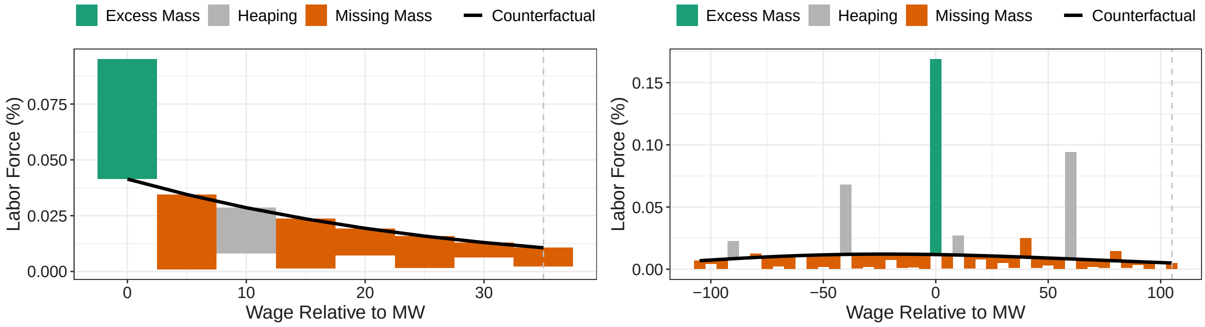


Figure B.1: "Local Stability" of Width of Missing Mass (2003)

Notes: The figure shows the estimated $\Delta^{*}w$ as a function of the interval I in both sectors. Sample selection as described in Section 2.
Source: PNAD and RAIS (2003). Use this to return to Section 3.

Main Results. We estimate $\Delta^{*}w = 35$ in the formal sector and $\Delta^{*}w = 105$ in the informal sector. The moments used in our SMM estimation will be these values relative to the minimum wage, $\Delta^{*}w/w_{\text{min}}$. Figure B.2 displays the estimated excess masses and counterfactual distributions for the formal and informal sectors. We highlight that, even in this local measure, the excess bunching at the minimum in the informal sector is *larger* than that in the formal sector and comes from both below and above w_{min}.[43] Furthermore, the estimated counterfactual polynomial already accounts for a portion of the spike in the formal sector.



(a) Formal Sector

(b) Informal Sector

Figure B.2: Excess Mass at the Minimum Wage, Missing Masses and Counterfactual Distributions (2003)

Notes: This figure shows the excess mass at the minimum wage (MW) using the estimated $\Delta^{*}w$ in each sector. Counterfactual distributions calculated using the fitted polynomial, excluding dummies and round numbers fixed effects. "Excess Mass" equals the $\hat{\beta}_{w_{\text{min}}}$ estimate. "Missing Mass" corresponds to $\hat{\beta}_j$ other than $\hat{\beta}_{w_{\text{min}}}$. Sample selection as described in Section 2.
Source: PNAD (2003). Use this to return to Section 3.

Figure B.3 shows the results of our bootstrap exercise to calculate the standard error of $\Delta^{*}w$. The bootstrap distribution for the informal sector is bimodal at $\{75, 105\}$, which aligns with the results of Figure B.1 for other intervals I. As a robustness exercise, Section 7.1 estimates the main specification of our model with the width of missing mass of the informal sector being 75.

---

[43] Reyes (2024) and Dube, Manning, and Naidu (2025) find widths of around 6–8% around prominent round numbers, which have less bunching than the minimum wage in the Brazilian setting.

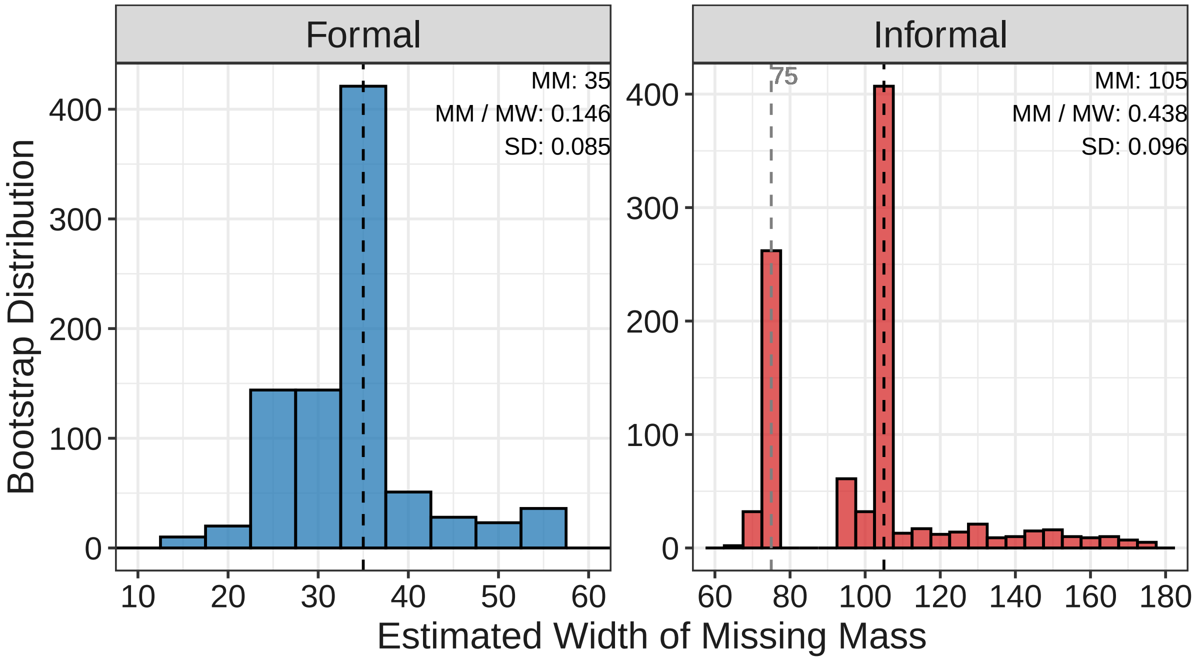


Figure B.3: Bootstrap Distribution of $\Delta^* w$ (2003)

Notes: The figure shows the histogram of the bootstrap distribution $\{\Delta^{*r} w\}_{r=1}^{R}$ with $R = 1,000$ residual bootstrap replications. "MM" = missing mass, "MW" = minimum wage, "SD" = standard deviation of $\Delta w^*/w_{\min}$. Dashed black lines highlight the estimated $\Delta^* w$ in the original sample. Sample selection as described in Section 2.
Source: PNAD (2003). Use this to return to Section 3.

Robustness. Following Dube, Manning, and Naidu (2025), we assess the sensitivity of our results to the degree of the polynomial used to approximate the counterfactual distribution. We consider $K \in \{4, 5, 6, 7\}$ and redo the "local stability" approach to find I in each case. Results for 2003 are in Table B.1, while those for the pooled 2003–2012 distribution without heaping controls are in Table B.2. Standard errors are calculated as the standard deviation of the residual bootstrap distribution with $R = 1,000$ replications.

Although I varies with the choice of K, the measures of interest – especially the widths of missing masses for 2003 – remain stable across specifications. For the 2003–2012 case, the widths are not stable for $K = 4$, and so we omit their results. We note that the excess and missing masses are not precisely estimated, with relatively large standard errors, particularly for the missing masses in the formal sector.[44]

Notably, in most cases the iterative procedure used to calculate $\Delta^* w$ yields a MM that considerably exceeds EM, which may lead to an overestimation of $\Delta^* w$. To address this potential issue, we conduct an alternative estimation for 2003 in which we choose $\Delta^* w$ to minimize $|EM - MM|$, instead of stopping at the first $\Delta^* w$ where $EM \leq MM$, and redo the "local stability" approach. The results (not shown for brevity) confirm the estimated width of the missing mass in the formal sector and generate a value of $\Delta^* w = 75$ in the informal sector, which will be used as a robustness exercise in Section 7.1.

---

[44] Dube, Manning, and Naidu (2025, Tables 1, C.3) also have relatively high standard errors, particularly for the missing mass estimates.

Table B.1: Robustness for Excess Mass, Missing Masses, and Width of Missing Masses (2003)

	Formal				Informal			
	$K=4$	$K=5$	$K=6$	$K=7$	$K=4$	$K=5$	$K=6$	$K=7$
Interval I	220	310	400	400	220	310	400	400
Excess Mass EM	0.04	0.05	0.05	0.04	0.16	0.15	0.16	0.16
	(0.02)	(0.03)	(0.02)	(0.07)	(0.00)	(0.00)	(0.00)	(0.00)
Missing Mass Below MMB	—	—	—	—	0.14	0.17	0.15	0.14
					(0.06)	(0.06)	(0.07)	(0.07)
Missing Mass Above MMA	0.12	0.10	0.10	0.09	0.13	0.12	0.09	0.10
	(0.10)	(0.13)	(0.13)	(0.13)	(0.05)	(0.04)	(0.04)	(0.04)
Missing Mass MM	0.12	0.10	0.10	0.09	0.27	0.29	0.23	0.24
	(0.10)	(0.13)	(0.13)	(0.13)	(0.08)	(0.08)	(0.09)	(0.09)
Width $\Delta^* w$	35	35	35	30	105	105	105	105
	(12.5)	(17.9)	(18.0)	(14.9)	(17.8)	(20.3)	(23.0)	(19.9)

Notes: This table shows the value for excess masses (EM), missing masses (MM) and width of missing masses ($\Delta^* w$) across different polynomial degrees K used to approximate the counterfactual distributions. Our main specification uses $K = 6$ and $b = 5$. For each K, we redo the "local stability" analysis to find I. Counterfactual distributions calculated using the fitted polynomial, excluding dummies and round numbers fixed effects. Sample selection as described in Section 2. Standard errors in parentheses calculated as the standard deviation of the residual bootstrap distribution, with $R = 1{,}000$ replications.
Source: PNAD (2003). Use this to return to Section 3.

Table B.2: Robustness for Excess Mass, Missing Masses, and Width of Missing Masses (2003–2012)

	Formal				Informal			
	$K=4$	$K=5$	$K=6$	$K=7$	$K=4$	$K=5$	$K=6$	$K=7$
Interval I	—	325	325	325	—	325	325	325
Excess Mass EM	—	0.09	0.08	0.02	—	0.17	0.16	0.17
		(0.06)	(0.03)	(0.05)		(0.00)	(0.00)	(0.00)
Missing Mass Below MMB	—	—	—	—	—	0.14	0.14	0.14
						(0.10)	(0.06)	(0.11)
Missing Mass Above MMA	—	0.13	0.10	0.21	—	0.06	0.09	0.08
		(0.17)	(0.15)	(0.14)		(0.08)	(0.07)	(0.10)
Missing Mass MM	—	0.13	0.10	0.21	—	0.20	0.23	0.22
		(0.17)	(0.15)	(0.14)		(0.12)	(0.11)	(0.13)
Width $\Delta^* w$	—	75	30	30	—	160	120	120
		(24.8)	(15.7)	(11.7)		(31.1)	(26.2)	(23.7)

Notes: This table shows the value for excess masses (EM), missing masses (MM) and width of missing masses ($\Delta^* w$) across different polynomial degrees K used to approximate the counterfactual distributions. Our main specification uses $K = 6$ and $b = 5$. For each K, we redo the "local stability" analysis to find I. Counterfactual distributions calculated using the fitted polynomial. "—" means that the estimated width was not stable in any interval $I \leq 500$. Standard errors in parentheses calculated as the standard deviation of the residual bootstrap distribution, with $R = 1{,}000$ replications. Sample selection as described in Section 2.
Source: PNAD (2003–2012). Use this to return to Section 3.

Table B.3 and Table B.4 report results using a binwidth of $b = 10$. Although a larger binwidth reduces granularity, it also mitigates sensitivity to reporting error, particularly for 2003. For the formal sector, the estimated widths of missing mass are consistent across values of K and closely mirror those obtained under $b = 5$, though they are less stable across intervals I (not shown for brevity). For the informal sector, estimates are highly stable in 2003 and consistent with the $b = 5$ case. $K = 4$ and $K = 5$ do not generate stable widths for the 2003–2012 distributions, so we omit the results. Overall, standard errors for the excess and missing masses are higher than those obtained with $b = 5$.

Table B.3: Robustness for Excess Mass, Missing Masses, and Width of Missing Masses (2003) – $b = 10$

	Formal				Informal			
	$K = 4$	$K = 5$	$K = 6$	$K = 7$	$K = 4$	$K = 5$	$K = 6$	$K = 7$
Interval I	220	310	370	480	220	310	370	480
Excess Mass EM	0.02	0.03	0.02	0.03	0.14	0.14	0.15	0.13
	(0.07)	(0.11)	(0.13)	(0.11)	(0.01)	(0.01)	(0.01)	(0.01)
Missing Mass Below MMB	—	—	—	—	0.11	0.15	0.12	0.26
					(0.16)	(0.06)	(0.10)	(0.07)
Missing Mass Above MMA	0.04	0.04	0.04	0.03	0.10	0.10	0.07	0.12
	(0.13)	(0.18)	(0.18)	(0.18)	(0.18)	(0.05)	(0.06)	(0.03)
Missing Mass MM	0.04	0.04	0.04	0.03	0.22	0.25	0.18	0.39
	(0.13)	(0.18)	(0.18)	(0.18)	(0.09)	(0.10)	(0.15)	(0.09)
Width $\Delta^* w$	30	30	30	30	100	100	100	110
	(15.0)	(20.7)	(18.9)	(23.3)	(13.5)	(14.2)	(18.0)	(7.7)

Notes: This table shows the value for excess masses (EM), missing masses (MM) and width of missing masses ($\Delta^* w$) across different polynomial degrees K used to approximate the counterfactual distributions. Our main specification uses $K = 6$ and $b = 5$. For each K, we redo the "local stability" analysis to find I. Counterfactual distributions calculated using the fitted polynomial, excluding dummies and round numbers fixed effects. Standard errors in parentheses calculated as the standard deviation of the residual bootstrap distribution, with $R = 1,000$ replications. Sample selection as described in Section 2.
Source: PNAD (2003). Use this to return to Section 3.

Table B.4: Robustness for Excess Mass, Missing Masses, and Width of Missing Masses (2003–2012) – $b = 10$

	Formal				Informal			
	$K = 4$	$K = 5$	$K = 6$	$K = 7$	$K = 4$	$K = 5$	$K = 6$	$K = 7$
Interval I	—	—	330	400	—	—	330	400
Excess Mass EM	—	—	0.06	0.02	—	—	0.16	0.16
			(0.10)	(0.09)			(0.01)	(0.01)
Missing Mass Below MMB	—	—	—	—	—	—	0.16	0.14
							(0.08)	(0.12)
Missing Mass Above MMA	—	—	0.06	0.09	—	—	0.09	0.06
			(0.21)	(0.17)			(0.08)	(0.10)
Missing Mass MM	—	—	0.06	0.09	—	—	0.25	0.20
			(0.21)	(0.17)			(0.10)	(0.14)
Width $\Delta^* w$	—	—	30	30	—	—	120	120
			(19.4)	(16.2)			(20.1)	(26.0)

Notes: This table shows the value for excess masses (EM), missing masses (MM) and width of missing masses ($\Delta^* w$) across different polynomial degrees K used to approximate the counterfactual distributions. Our main specification uses $K = 6$ and $b = 5$. For each K, we redo the "local stability" analysis to find I. Counterfactual distributions calculated using the fitted polynomial. Standard errors in parentheses calculated as the standard deviation of the residual bootstrap distribution, with $R = 1,000$ replications. "—" means that the estimated width was not stable in any interval $I \leq 500$. Sample selection as described in Section 2.
Source: PNAD (2003–2012). Use this to return to Section 3.

Alternative Counterfactual Distribution. Let $\tilde{G}_i(w)$ denote the "partial" counterfactual wage distribution absent intensive responses to the minimum wage in sector i, with $i = 1$ denoting the formal and $i = 2$ the informal sector. Until now, we have assumed that these distributions can be written as flexible polynomials, with the excess and missing masses relative to the observed distribution determined by the OLS coefficients. We now consider an alternative approach.

Specifically, we assume that the latent wage distribution in each sector i can be modeled as a mixture of two lognormals, the CDF of which we denote by $F_i(w)$. We choose this parametric form because of its flexibility and relative parsimony, as well as the fact that it does not require specifying bounds for its support. To be consistent with the "partial" counterfactual interpretation, we censor observations in the formal sector with wages below the minimum. In other words, we move all predicted mass below the minimum to it, which creates a spike in the formal sector even absent any missing masses. Formally, the partial counterfactual CDF is given by:

$$\tilde{G}_1(w) = \begin{cases} 0, & w < w_{\min} \\ F_1(w), & w \geq w_{\min} \end{cases} \qquad \text{and} \qquad \tilde{G}_2(w) = F_2(w)$$

To fit the observed distribution $G_i(w)$, we also allow for the existence of missing masses with width Δw_i and height h_i. It will be useful to define the missing mass intervals:

$$\mathscr{W}_1 = (w_{\min}, w_{\min} + \Delta w_1] \qquad \mathscr{W}_2 = [w_{\min} - \Delta w_2, w_{\min} + \Delta w_2] \setminus \{w_{\min}\}$$

where we consider the existence of a missing mass only above $w_{\min}$ in the formal sector, but consider a symmetric region around the minimum in the informal sector. Then, we assume that the observed wage CDF can be written as:

$$G_i(w) = \tilde{G}_i(w) + (1 - h_i)\mathbb{P}(w \in \mathscr{W}_i) \left[\mathbb{1}\{w \geq w_{\min}\} - \tilde{G}_i(w)(w \mid w \in \mathscr{W}_i)\right] \tag{B.2}$$

where $\tilde{G}_i(w)(w \mid w \in \mathscr{W}_i)$ is a conditional CDF. In words, we assume that the observed distribution equals the latent, except around the minimum wage. For $w \in \mathscr{W}_i$, the distribution has missing masses. Additionally, there is a mass point at the minimum wage: as in our model, we assume that the missing mass around the minimum wage is transferred to it, contributing to the spike. Note the similarity of Equation B.2 with Proposition 3.

To build intuition, the wage PDFs at $w \neq w_{\min}$ are $g_i(w) = \tilde{g}_i(w)\left[1 - (1 - h_i)\mathbb{1}\{w \in \mathscr{W}_i\}\right]$, with missing masses $\vartheta_i(w) = \tilde{g}_i(w)(1 - h_i)$ for $w \in \mathscr{W}_i$ and 0 otherwise. Then, the minimum wage spikes are given by:

$$\begin{aligned} v_1 &= \tilde{G}_1(w_{\min}) + (1 - h_1)\left[\tilde{G}_1(w_{\min} + \Delta w_1) - \tilde{G}_1(w_{\min})\right] \\ v_2 &= (1 - h_2)\left[\tilde{G}_2(w_{\min} + \Delta w_2) - \tilde{G}_2(w_{\min} - \Delta w_2)\right] \end{aligned} \tag{B.3}$$

As in our model of Section 5, there can be two sources of bunching in the formal sector: one due to jobs that would have paid wages below the minimum absent the bite of the policy, and another due to the excess mass transferred from the missing mass above $w_{\min}$. The spike can thus be matched through censoring alone, excess mass alone, or a combination of the two. In the informal sector, the entirety of the spike comes from the missing mass around $w_{\min}$.

Relative to the standard bunching method, there is a trade-off by assuming a parametric form – both to the latent distribution and to the shape of the missing mass, imposing that it is a uniform proportional reduction of the latent density –, but, while the bunching approach uses an iterative procedure to impose that the missing mass has to be equal to or greater than the excess mass at the spike, this approach imposes equality of the two.[45] Note that the shape restriction of the missing mass in this alternative procedure is consistent with our model, where the missing mass is proportional to the share of Type-B firms (Equation 9).

In this specification, each $F_i(w)$ has five parameters: the means and standard deviations of the two lognormals, and the mixture weight. We also have two parameters governing the height and width of the missing mass. To estimate them, we use a minimum distance procedure to estimate the seven parameters of $G_i(w)$ in each sector, targeting the minimum wage spikes and the percentiles $p \in \{5, 10, 15, \cdots, 80\}$.[46]

Intuitively, the percentiles outside the missing mass region pin down the shape of the latent distribution $F_i(w)$, which in turn predicts what the density should look like within $\mathscr{W}_i$. The discrepancy between the predicted and observed density in that region identifies the missing mass parameters h_i and Δw_i. In the formal sector, the share of the spike due to censoring then follows residually: it equals the observed spike minus the excess mass transferred from the missing mass region. Thus, the decomposition is disciplined by the fit of the distribution above $w_{\min}$, though it does require both extrapolation and that the parametric form be locally accurate within $\mathscr{W}_i$. Note, however, that this approach is silent about the economic forces behind censoring in the formal sector: the mass attributed to censoring is not a counterfactual claim about what would happen to these workers absent the minimum wage, but a statistical device for decomposing the observed spike.[47] If some of these workers would in fact be unemployed absent the policy, the censoring share would be overstated and the excess mass share understated, but the total spike and the estimated missing masses – which are the objects of interest – would be largely unaffected, as they are pinned down by the fit on the observed distribution. Moreover, if disemployment effects are relevant, this would only increase the missing mass share of the spike, requiring a larger missing mass and further corroborating the existence of these regions.

For this method, we express wages as $w/w_{\min}$ to accommodate the use of various years; we do not use the real $w - w_{\min}$ because it is incompatible with the support of the lognormals. This is only relevant for the pooled 2003–2012 wage distributions, as dividing by $w_{\min}$ does not affect

---

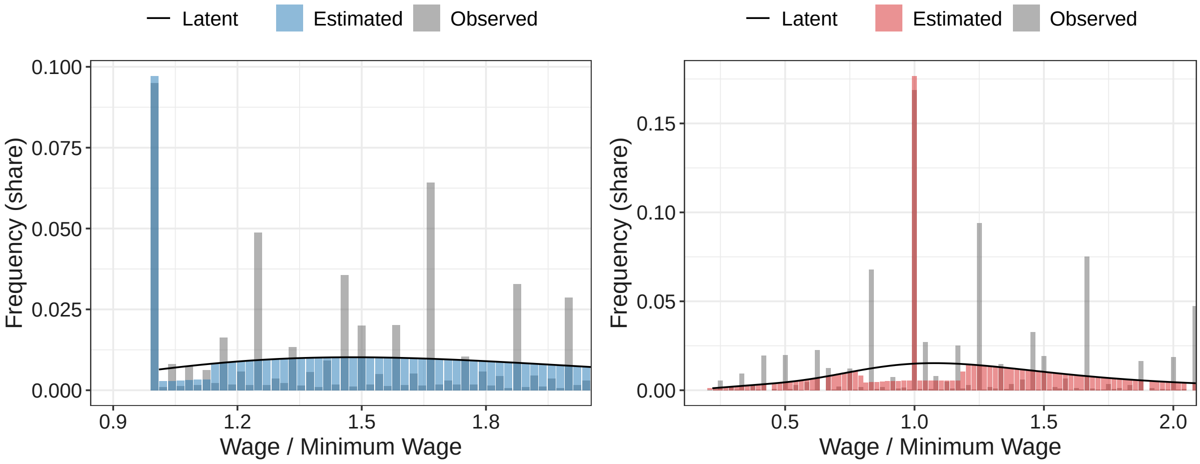
[45] Additionally, the bunching approach is estimated on the binned data (Equation B.1), while this method uses the individual microdata directly.

[46] As in the estimation of the model, we exclude percentiles that coincide with the minimum wage. We use an identity matrix, minimizing the square of the percentage differences between observed and fitted moments.

[47] Jales and Yu (2024) also use a lognormal parametric form to extrapolate wages below the minimum, but account for job loss based on a logit specification in a second step.

the fit of single-year estimations. We also fit yearly distributions on real wages for each year between 2003 and 2012 and then aggregate the results to form a pooled distribution: results are virtually the same as those using $w/w_{\min}$ and are omitted for brevity.

Figure B.4 and Figure B.5 show the estimated fit for the 2003 and pooled 2003–2012 wage distributions, respectively. The exercise confirms the existence of missing masses, although in 2003–2012 we estimate a notably smaller width in the formal sector and a nearly disconnected support in the informal sector.

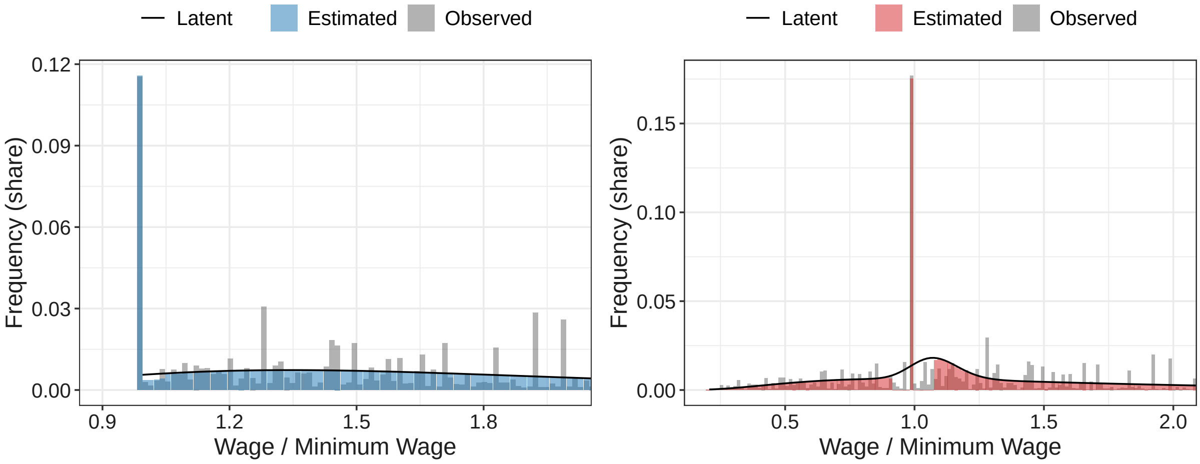


(a) Formal Sector

(b) Informal Sector

Figure B.4: Fit of the Lognormal Mixture Distributions with Missing Masses (2003)

Notes: This figure shows the fit of the mixture of lognormal distributions with excess masses at the minimum, missing mass around it, and censoring in the formal sector. Binwidth for histograms: 2.08% of $w_{\min}$ (R$ 5 of R$ 240). Sample selection as described in Section 2.
Source: PNAD (2003). Use this to return to Section 3.



(a) Formal Sector

(b) Informal Sector

Figure B.5: Fit of the Lognormal Mixture Distributions with Missing Masses (2003–2012)

Notes: This figure shows the fit of the mixture of lognormal distributions with excess masses at the minimum, missing mass around it, and censoring in the formal sector. Binwidth for histograms: 1.33% of $w_{\min}$ (average of R$ 5 over each year's minimum wage). Sample selection as described in Section 2.
Source: PNAD (2003–2012). Use this to return to Section 3.

Table B.5 shows the estimated missing mass widths in each sector and method for 2003, the year used in the estimation of our model. While the widths are the same in the formal sector, we estimate a smaller width in the informal sector. As mentioned previously, we conduct robustness exercises in our model with a smaller width of missing mass in the informal sector in Section 7.1.

Table B.5: Robustness of Width of Missing Masses – Alternative Counterfactual Distributions (2003)

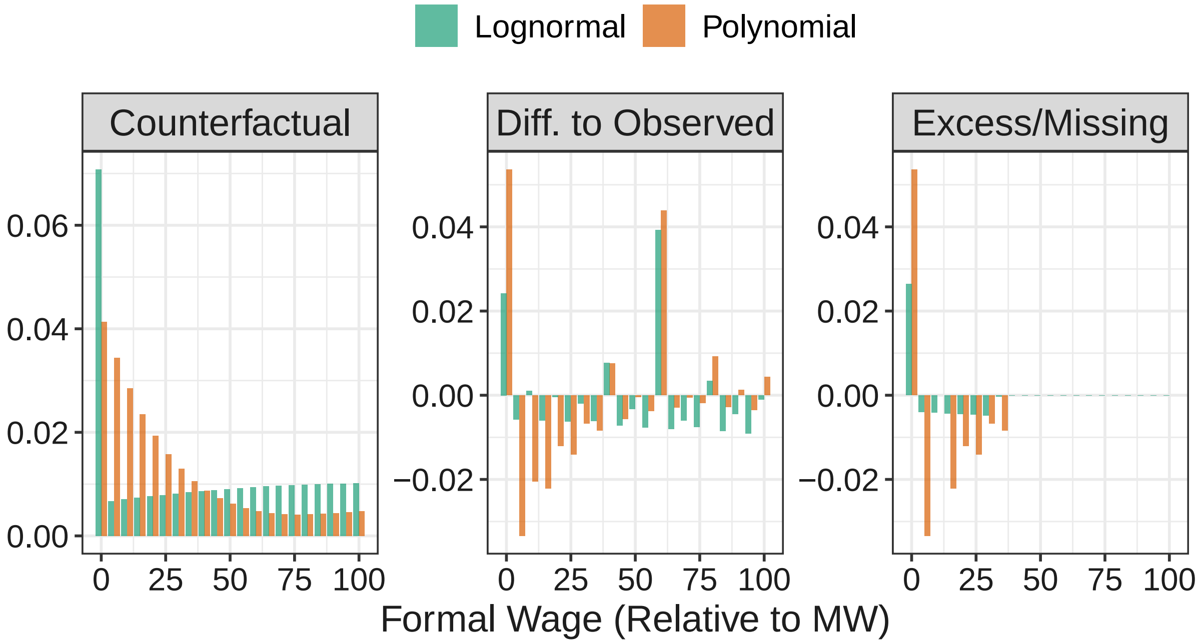
	Formal		Informal	
	Polynomial	Lognormal	Polynomial	Lognormal
Width of Missing Mass	35.0	35.3	105.0	47.0

Notes: This table shows the value for the estimated width of missing masses ($\Delta^* w$) across different methods to estimate the "partial" counterfactual distributions. In the lognormal method, we estimate the distribution on w/w_{min} and convert the estimated widths back to nominal terms. Sample selection as described in Section 2.
Source: PNAD (2003). Use this to return to Section 3.

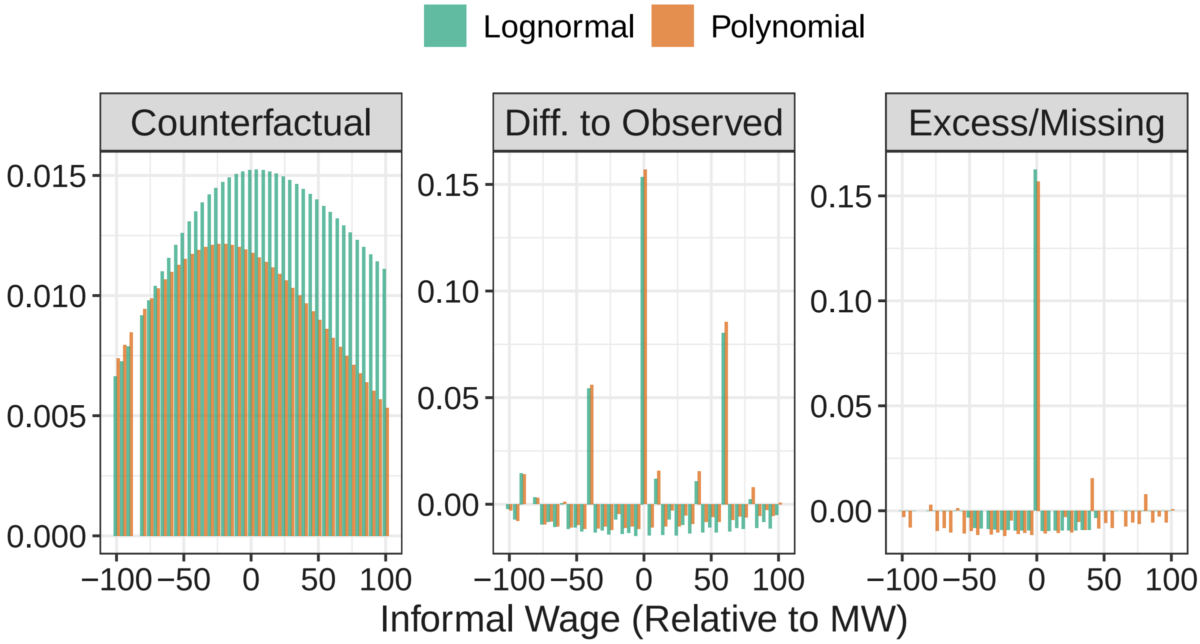
Figure B.6 compares the partial counterfactual distributions and estimated excess and missing masses of each method for 2003. In the formal sector, the lognormal fit estimates a lower excess and missing mass – which accounts for 2.4 p.p. of the predicted 9.7% in the minimum wage bin –, indicating that the portion of the spike due to censoring is larger in this method. In turn, the bunching approach estimates that the excess mass coming from above represents 5.3 p.p. of the observed 9.5% of the population in the minimum wage bin.

Interestingly, the estimated heights of the excess and missing masses in the informal sector are very similar across methods, despite the larger width in the bunching approach. This is because the bunching method imposes an inequality restriction, while the lognormal fit imposes equality between the missing and excess masses: indeed, Table B.1 shows that the estimated missing mass in the bunching approach is larger than the excess mass at the minimum.

Overall, this alternative exercise confirms the existence of missing masses around the minimum wage in both sectors, although with varying magnitudes in the informal sector. Both methods – Equation B.1 and Equation B.2 – show there exists an excess mass at the minimum wage in the formal sector beyond that generated by purely censoring. Furthermore, both approaches can successfully match the minimum wage spike in the informal sector.



(a) Formal Sector



(b) Informal Sector

Figure B.6: Comparison of Counterfactual Distributions and Missing Masses (2003)

Notes: This figure compares the estimated counterfactual distributions and missing masses across methods. "Polynomial" refers to the standard bunching procedure (Equation B.1), while "LogNormal" refers to the parametric fit (Equation B.2). "Counterfactuals" refers to the partial counterfactual of each method: either the polynomial term $\sum_{k=0}^{K} \alpha_k w^k$ or the (censored) mixture of lognormals $\tilde{G}_i(w)$. "Diff. to Observed" is the difference between the observed distributions in the data and the estimated counterfactual distributions. "Excess/Missing" is the estimated excess at the minimum wage or missing mass around it. In the polynomial case, it is given by the coefficients β_j and coincides with the observed differences within the estimated missing mass region. In the lognormal case, it is given by $(1-h_i)\tilde{G}_i(w)$ for wages within the estimated missing mass region. Sample selection as described in Section 2.
Source: PNAD (2003). Use this to return to Section 3.

C Wage Measurement and Formal Labor Costs

This appendix discusses the wage measurement in the data we use, building on Haanwinckel and Soares (2021, Appendix A) and Derenoncourt et al. (2025, Appendix A4).

Wage Measurement. Wages in the household surveys we use are reported in monthly, nominal and gross terms. Workers are asked to report their "habitual" wage, excluding one-off remunerations (such as the thirteenth salary) or discounts. What we refer to as the *minimum wage spike* is the share of workers earning exactly the gross, nominal and monthly minimum wage.

In the formal sector, workers report wages before their social security contributions and income tax, but after the employer social contribution – which is consistent with the location of the minimum wage spike. Informal workers are not subject to such costs, so their reported salary is already their net, take-home wage. Haanwinckel and Soares (2021) and Finamor (2025) show that workers in Brazil and Chile, respectively, prefer formal jobs to informal ones paying the same salary even after considering the higher discounts in the former, so compensating differentials still exist net of taxes and social security contributions.

Employer Contributions. The model captures all non-wage employer contributions through τ in the formal sector profit function. Therefore, this "payroll tax" parameter must contain all costs a formal employer has to pay, but an informal one does not. We follow Haanwinckel and Soares (2021, Table A1) and fix $\tau = 0.7206$, as they also focus their analysis on the last quarter of 2003. This number captures worker benefits (thirteenth salary, paid vacation), contributions to social security, and expected severance and dismissal costs.

Worker Contributions and Benefits. The model captures worker income taxes and social security contributions through the parameter τ_w. Both income taxes and social security contributions have progressively increasing rates depending on the salary; for simplicity, we consider a single rate for all wages. We follow Ulyssea (2010) and fix $\tau_w = 0.10$.

Additional formal benefits apply in the case of dismissal: severance, paid as a percentage s of the wage, and unemployment insurance $UI(w)$, which depends on the worker's wage and on the length of the employment spell. In reality, severance is paid as 40% of the balance in the worker's social security fund (FGTS), but this variable is not observable in the data and so we adopt the approximation of a percentage of the salary. We take from the implied values in Haanwinckel and Soares (2021, Table A2) and fix $s = 0.7936$.[48]

The values of unemployment insurance $UI(w)$ – which, as stated in the main text, is assumed to be paid upfront – are endogenously determined using the model's equilibrium wage

---

[48] The value corresponds to the 2003 "Low wage" column. The probability of dismissal can be inferred from panel A: $26.52/(508.84+42.40+14.13) \approx 4.69\%$. The severance payment as a percentage of the wage is thus $18.94/508.84/0.0469 \approx 79.36\%$. The value is noticeably larger than Meghir, Narita, and Robin (2015) (12.5%), who consider the same rate in the employer contribution and in the worker's value function.

offer distribution and the policy's wage schedules for the last quarter of 2003. UI is paid for three to five months depending on the duration of the job spell; for simplicity, we assume a duration of four months, which is the value for those employed for one year. We then calculate $UI(w)$ for every formal firm and take the mean to get $\mathbb{E}[UI(w)]$. As $\mathbb{E}[w_1]$, the expectation in $\mathbb{E}[UI]$ is taken with respect to F_1, and so is not weighted by the number of workers in each firm. Finally, note that UI and s are not subject to income taxes under Brazilian law.

In practical terms, τ_w, s and $UI(w)$, as well as the differential disutilities of effort e_i and job separation rates q_i, are captured in Λ_i and η_i in our parametrization of the labor supply curve, $i = 1,2$. Therefore, they only affect the calculation of the value of unemployment V^U in our counterfactuals.

D Model Proofs and Additional Details

D.1 Post-Entry Productivity Distribution

This appendix details the expressions for the post-entry productivity distribution of Section 5.1.4. Let $h(\theta)$ and $\gamma(p \mid \theta)$ denote the densities associated with $H(\theta)$ and $\Gamma(p \mid \theta)$, respectively. The post-entry, unconditional distributions will be given by the following truncated densities:

$$\tilde{\gamma}_2(p) = \frac{1}{H(\underline{\theta}_1) - H(\underline{\theta}_2)} \int_{\underline{\theta}_2}^{\underline{\theta}_1} \gamma(p \mid \theta)\, d\theta \quad \text{and} \quad \tilde{\gamma}_1(p) = \frac{1}{1 - H(\underline{\theta}_1)} \int_{\underline{\theta}_1}^{\infty} \gamma(p \mid \theta)\, d\theta$$

However, after entry in sector $i = 1,2$, firms may get a bad draw of $p \mid \theta$ such that $\pi_i^A(p) < 0$, and so they exit the market. Let $\underline{p}_i = \inf\left\{p : \pi_i^A(p) \geq 0\right\}$ denote the lowest productivity that attains positive profits in sector $i = 1,2$. The post-entry density among successful entrants in sector $i = 1,2$ will be given by:

$$\gamma_i(p) = \frac{\tilde{\gamma}_i(p)}{1 - \tilde{\Gamma}_i\left(\underline{p}_i\right)} \quad \text{if } p \geq \underline{p}_i \text{ and 0 otherwise,} \tag{D.1}$$

where $\tilde{\Gamma}_i$ is the CDF associated with $\tilde{\gamma}_i$. $\Gamma_1(p)$ and $\Gamma_2(p)$ are given by integrating Equation D.1: $\Gamma_i(p) = \int_{\underline{p}_i}^{p} \gamma_i(x)\, dx$. As our primitives, these distributions are absolutely continuous in their support $[\underline{p}_i, \infty)$.

Finally, the measure of successful entrants in each sector $i = 1,2$ will be given by:

$$\begin{aligned} N_1 &= \left[1 - \tilde{\Gamma}_1\left(\underline{p}_1\right)\right] [1 - H(\underline{\theta}_1)] N \\ N_2 &= \left[1 - \tilde{\Gamma}_2\left(\underline{p}_2\right)\right] [H(\underline{\theta}_1) - H(\underline{\theta}_2)] N \end{aligned}$$

where $[1 - H(\underline{\theta}_1)]N$ and $[H(\underline{\theta}_1) - H(\underline{\theta}_2)]N$ are the masses of potential entrants that choose the formal or informal sector, respectively. However, only a fraction $1 - \tilde{\Gamma}_i\left(\underline{p}_i\right)$ attain positive profits in each sector $i = 1,2$. The expression for the informal sector can omit this term, as we do not have fixed costs of operations, but we include it for completeness.

D.2 Microfoundation of the Labor Supply Curve

This appendix presents the microfoundation of the positive labor supply curve using the efficiency wage framework of Rebitzer and Taylor (1995).

D.2.1 Setting

There is a measure M of homogeneous, risk-neutral, infinitely lived workers, who discount the future at a rate $1/(1+r)$, $r \geq 0$. Following the efficiency wage model of Shapiro and Stiglitz (1984), all workers dislike exerting effort, but enjoy consuming goods, and have an additively separable utility function. Individuals can neither lend nor borrow, so they consume their entire income in each period and maximize their expected discounted lifetime utility.[49]

Workers can be in one of three sectors i: unemployment ($i = 0$), formal ($i = 1$), or informal ($i = 2$). Formal workers are subject to the minimum wage and are eligible for unemployment insurance, but must pay income taxes and social security contributions. Informal workers avoid these costs, but forgo these benefits.

Workers only produce output when exerting effort: those caught not doing so are fired at the end of the period and transition into unemployment. Importantly, monitoring by firms is imperfect and they cannot fully observe workers' effort, so there is a moral hazard problem. For simplicity, workers' choice of effort is discrete: they provide either no effort or a positive amount $e_i > 0$, which we allow to vary by sector to capture the greater flexibility associated with the informal sector (e.g., Rauch (1991)). Workers in the unemployment state are assumed to not bear this effort cost and have a flow utility value of b, capturing the value of leisure, home production, as well as the disutility of being unemployed. Alternatively, one may think of unemployment as a broader outside option, including self-employment; in this interpretation, b includes the revenues from this activity net of effort costs.[50]

In their decision to exert effort or not, workers face a trade-off: effort entails a utility cost of e_i, but lowers the probability of job loss.[51] Let $q_i \in (0,1)$, $i = 1,2$, be the exogenous probabilities at which jobs are terminated, which vary by sector since informal turnover is higher (Gomes, Iachan, and Santos 2020; Engbom et al. 2022). Workers not exerting effort – or "shirking" – are detected with probability $D_i \in (0,1)$, and so face a total separation probability of $q_i + (1-q_i)D_i$; we assume that the exogenous termination happens immediately before the dismissal by shirking.

In the model of Shapiro and Stiglitz (1984), D_i is constant. We follow Rebitzer and Taylor (1995) and assume that the detection technology is strictly decreasing in the amount of firm

---

[49] We assume worker homogeneity mainly for tractability. This makes our model unfit to answer questions about redistribution (Hurst et al. 2022; Berger, Herkenhoff, and Mongey 2025), but suited to explain inequality since it generates wage dispersion. Additionally, we are unable to capture changes in workforce composition, which matter for explaining informality trends in Brazil (Haanwinckel and Soares 2021).

[50] See Narita (2020), Amodio, Medina, and Morlacco (2024), and Finamor (2025) for self-employment models.

[51] Rebitzer (1995) and Cai, Luo, and Wang (2025) provide empirical evidence in support of efficiency wage theory, showing that higher pay reduces quit rates and is associated with a lower degree of monitoring by managers.

employment ℓ, as firms have to observe a larger number of workers. This is the fundamental assumption of our microfoundation and gives rise to the upward-sloping labor supply curves.

Assumption 1 (Monitoring Technology)**.** *Let ℓ be the size of the firm's labor force. For $i = 1, 2$, the probability of shirking detection $D_i : \mathbb{R}_+ \to [0,1]$ has the following properties:*

1. *$D_i(\ell)$ is twice continuously differentiable and strictly decreasing.*
2. *The inverse function D_i^{-1} is log-concave in the interval $(0,1)$.*

One may interpret this as firms having a fixed amount of monitoring resources (e.g., managers) available, which they must allocate across all of their workers. More broadly, one may think that firms hire managers at a lower rate than production workers, so each manager must monitor a higher number of employees when the firm grows. Therefore, the larger the firm's labor force size, the lower the amount of monitoring per worker.[52,53]

The subscript i in $D_i(\ell)$ reflects that this technology differs by sector due to differences in separation rates and costs of effort. It is worth noting that seminal papers on efficiency wages in dual-sector economies such as Bulow and Summers (1986) and Albrecht and Vroman (1992) assume that effort is perfectly observable in the lower-paying sector, so that firms pay workers' reservation wages. As argued by Ulyssea (2010), perfect competition does not adequately describe the functioning of the informal sector, so we abstract from this setup.

D.2.2 Value Functions

Formal Sector. Formal workers benefit from labor regulations and lower separation rates, so jobs paying the same wage can be valued differently across sectors in the model. The value of ***N***ot shirking in the formal sector ($i = 1$) in its Bellman form is:

$$V_1^N(w) = (1-\tau_w)w - e_1 + \frac{1}{1+r}\Big\{(1-q_1)V_1^N(w) + q_1\left[V^U + UI + s\cdot w\right]\Big\}, \tag{D.2}$$

where $\tau_w \in [0,1)$ captures income taxes and social security contributions paid by workers, V^U is the value of unemployment, $UI = UI(w)$ is the unemployment insurance and $s \in (0,1)$ is the severance rate. For simplicity, we assume UI is paid upfront and is not included in V^U, so unemployment duration and previous sector of employment are not state variables (Meghir, Narita, and Robin 2015; Haanwinckel and Soares 2021). Importantly, we abstract from job-to-job transitions as these models generally do not allow for equilibrium wage offer distributions with mass points, as we detail in Appendix E.

---

[52] Although we treat this technology as exogenous, Piyapromdee (2018) shows that the trade-off between monitoring and firm size emerges endogenously in a wage-posting, on-the-job search model where firms face a monitoring cost per worker. However, her model does not generate equilibrium wage distributions with mass points.

[53] Differentiability and log-concavity of D_i^{-1} are sufficient regularization conditions that ensure the firm's problem has a unique solution (Kline 2025). Log-concavity of D_i^{-1} means that the function cannot be "too convex". It does not imply log-concavity of D_i; using the rules of inverse functions, log-concavity of D_i^{-1} is assured if $D_i'' \cdot D_i^{-1} \leq -D_i'$. Since $D_i' < 0$, the RHS is positive, and so this condition also means that D_i cannot be "too convex".

The value of *S*hirking has a similar form, but disregarding the effort cost and including the probability of detection:

$$V_1^S(w) = (1-\tau_w)w + \frac{1}{1+r}\Big\{(1-q_1)(1-D_1(\ell))V_1^S(w) + q_1\left[V^U + UI + s\cdot w\right] + (1-q_1)D_1(\ell)V^U\Big\} \tag{D.3}$$

Workers are paid at the beginning of the period. They are not fired with probability $(1-q_1)(1-D_1(\ell))$, and transition into unemployment with complementary probability. Workers may be fired for exogenous reasons, which happens with probability q_1, or because they were caught shirking, with probability $(1-q_1)D_1(\ell)$; we assume that exogenous termination happens immediately before the firing by shirking, with both occurring at the end of the period.

By Brazilian law, formal workers fired for justified reasons ("*justa causa*") lose some of their benefits, including severance and unemployment insurance. In Equation D.3, we assume that job termination for shirking falls in this category. We additionally assume that workers cannot appeal this process in labor courts and that firms cannot see the firing history of their job applicants, abstracting from adverse selection problems as in Shapiro and Stiglitz (1984).[54]

The only choice employed workers make is whether to exert effort, which is discrete in our model. Since they are homogeneous and do not generate output when not exerting effort, if one worker chooses to shirk, all do and there is no output. Therefore, we must have the following *no shirking condition* (NSC):

$$V_1^N(w) \geq V_1^S(w) \quad \text{for all } w \tag{D.4}$$

If the firm could perfectly observe effort, it would instantly fire all shirkers. As it cannot, the firm only detects them with probability $D_1(\ell)$, which, in principle, allows shirkers to exist. To solve this moral hazard problem, it must incentivize effort through higher wages. This increases the "penalty" for job loss, which occurs with higher probability among shirkers.

Informal Sector. The expressions in the informal sector take a similar form, but without wage taxes or unemployment benefits:

$$V_2^N(w) = w - e_2 + \frac{1}{1+r}\Big\{(1-q_2)V_2^N(w) + q_2V^U\Big\} \tag{D.5}$$

$$V_2^S(w) = w + \frac{1}{1+r}\Big\{(1-q_2)(1-D_2(\ell))V_2^S(w) + [1-(1-q_2)(1-D_2(\ell))]V^U\Big\} \tag{D.6}$$

Since we do not allow for informal workers to generate output when shirking or for informal firms to perfectly monitor their workers, here too a NSC arises:

$$V_2^N(w) \geq V_2^S(w) \quad \text{for all } w \tag{D.7}$$

---

[54] See Araujo, Ponczek, and Souza (2016) and Corbi et al. (2022) for models where employees can take firms to court. One could also consider a more general setting where a fraction $\tilde{q}_1 \in [0,1)$ of shirkers still receive formal unemployment benefits; the conclusions are essentially the same.

Unemployment. Unemployed workers apply for jobs in both sectors and find one with probability $\phi \in (0,1)$. Similarly to Haanwinckel and Soares (2021), we rule out unemployed workers receiving more than one offer per period to simplify the decision process. Given that the worker has found a job, it is in the formal sector with probability $\phi_1 \in (0,1)$ and from the informal one with probability $\phi_2 = 1 - \phi_1$. After the offer has arrived, its wage is randomly sampled from the distribution of wage offers in each sector, similarly to a random search process.

For ease of exposition, in what follows we take NSCs in Equations D.4 and D.7 as valid, so that $\max\{V_i^N(w), V_i^S(w)\} = V_i^N(w)$ for $i = 1,2$ and all w. Additionally, we assume all wage offers are acceptable, so that $V_i^N(\underline{w}) \geq V^U$ for $i = 1,2$. Both conditions will be true in equilibrium: no active firm can produce with a shirking labor force or without attracting any workers. With these considerations in mind, we can write the value of being unemployed as:

$$V^U = b + \frac{1}{1+r}\left\{\phi\left(\phi_1 \mathbb{E}\left[V_1^N(w)\right] + \phi_2 \mathbb{E}\left[V_2^N(w)\right]\right) + (1-\phi)V^U\right\} \tag{D.8}$$

Unemployed workers receive a flow value of b, capturing, for example, leisure and basic income. With probability $1-\phi$, they remain unemployed. With probability ϕ, they receive an offer; as all offers are acceptable, they transition into the corresponding sector. From the workers' perspective, ϕ is exogenous, but it is an endogenous object which depends on the unemployment rate and other primitives. As ϕ and $\mathbb{E}\left[V_i^N(w)\right]$, $i = 1,2$, are endogenous, V^U itself is an equilibrium object. We elaborate on this in Appendix D.3; for now, note that V^U is not a function of any specific w, only of the distribution of wages in both sectors.

As emphasized by Shapiro and Stiglitz (1984), there are neither search frictions nor vacancies in the model despite the similarities with the value functions of search frameworks (e.g., Rogerson, Shimer, and Wright (2005)). Workers have perfect information about all job opportunities, but it is not optimal for firms to hire them all. This is because there is a relationship between wages and labor productivity, which stems from imperfect effort monitoring. Firms must make holding a job and exerting effort more attractive than shirking; if there is no unemployment and shirkers are immediately rehired, the punishment for not exerting effort is essentially negligible. To solve this moral hazard problem, firms offer higher wages, which decreases their labor demand and creates unemployment. Since $V_i^N(\underline{w}) \geq V^U$ for $i = 1,2$, all unemployment is involuntary: workers would prefer to work at any wage rather than remain unemployed. However, if firms offer lower wages, workers cannot make a credible promise not to shirk. Since shirking leads to no output, it will not be optimal for firms to offer lower wages.[55]

---

[55] Following Shapiro and Stiglitz (1984), we only need to assume that the output of shirking is sufficiently low. See Albrecht and Axell (1984) for a model where firms find it optimal to hire some shirkers.

D.2.3 No Shirking Conditions and Labor Supply Curves

Proposition 4 derives wages such that NSCs in Equations D.4 and D.7 hold. Note that these conditions are binding: it is not optimal for firms to pay a higher wage than the one which makes equality hold, since it elicits no further effort from workers as it is a discrete choice. If firms offer lower wages, workers prefer to shirk and there is no output.

Proposition 4 (Efficiency Wages, proof)**.** *Let $A_i = r + q_i$ and $B_i = (1 - q_i)D_i(\ell)$ for $i = 1,2$. Workers have no incentive to shirk in the formal sector ($i = 1$) if:*

$$w(1+r)B_1\left[(1-\tau_w)+\frac{q_1\cdot s}{1+r}\right] = (1+r)e_1(A_1+B_1) - q_1 B_1 UI + rB_1 V^U$$

Workers in the informal sector ($i = 2$) have no incentive to shirk if:

$$w(1+r)B_2 = (1+r)e_2(A_2+B_2) + rB_2 V^U$$

The implied monitoring probabilities are:

$$\begin{aligned} D_1(\ell) &= \frac{e_1(r+q_1)}{(1-q_1)\left((1-\tau_w)w - e_1 - \frac{r}{1+r}V^U + \frac{q_1}{1+r}(UI + s\cdot w)\right)} \\ D_2(\ell) &= \frac{e_2(r+q_2)}{(1-q_2)\left(w - e_2 - \frac{r}{1+r}V^U\right)} \end{aligned} \tag{D.9}$$

Proposition 4 gives the efficiency wage expressions in each sector. By Assumption 1, $D_i(\ell)$ is invertible, and so we have a relationship between ℓ_i and w in each sector $i = 1,2$:

Proposition 5 (Labor Supply, proof)**.** *The labor supply curves $\ell_1(w)$ and $\ell_2(w)$ are given by:*

$$\begin{aligned} \ell_1(w) &= D_1^{-1}\left(\frac{e_1(r+q_1)}{(1-q_1)\left((1-\tau_w)w - e_1 - \frac{r}{1+r}V^U + \frac{q_1}{1+r}(UI + s\cdot w)\right)}\right) \\ \ell_2(w) &= D_2^{-1}\left(\frac{e_2(r+q_2)}{(1-q_2)\left(w - e_2 - \frac{r}{1+r}V^U\right)}\right) \end{aligned} \tag{D.10}$$

and are twice continuously differentiable, strictly increasing in w, and log-concave.

Intuitively, Proposition 5 shows that to attract more non-shirking workers, firms must share rents in the form of higher wages to compensate for the fall in detection probability as they grow larger. Alternatively, firms of higher productivity have “more to lose” when workers shirk, and so pay higher wages (Ramaswamy and Rowthorn 1991). The NSCs thus become *implicit upward-sloping labor supply curves*, which is the key insight of Rebitzer and Taylor (1995) and the defining characteristic of monopsony models.[56] Note that the labor supply curves are a function of V^U, and so are affected by average wages, unemployment, and informality.

Figure A.13 and Figure A.15 provide empirical support for an upward-sloping labor supply curve, showing that the probability of being a minimum wage worker decreases with firm size and that minimum wage employment in the formal sector is concentrated in smaller firms. Figure A.16 highlights that the correlation of firm size with wages in the Brazilian formal sector is positive even when controlling for individual, local, and firm characteristics.

D.2.4 Value of Unemployment

One of the key equilibrium objects in our model is the value of unemployment V^U. We are now ready to write it and the other value functions in terms of the primitives in our model; all expected values are taken with respect to the distributions of wage offers $F_1(w)$ and $F_2(w)$.

Proposition 6 (Value Functions, proof)**.** *The value functions of workers are:*

$$\begin{aligned} &\textit{Not Shirking: } \mathbb{E}\left[V_1^N(w)\right] = \frac{K_1 + q_1 V^U}{r+q_1} \qquad \mathbb{E}\left[V_2^N(w)\right] = \frac{K_2 + q_2 V^U}{r+q_2} \\ &\textit{Unemployment: } V^U = \frac{(1+r)b + \phi\left(\phi_1 \frac{K_1}{r+q_1} + \phi_2 \frac{K_2}{r+q_2}\right)}{r + \phi - \phi\left(\phi_1 \frac{q_1}{r+q_1} + \phi_2 \frac{q_2}{r+q_2}\right)} \end{aligned} \tag{D.11}$$

where $K_1 = (1+r)\left((1-\tau_w)\mathbb{E}[w_1] - e_1\right) + q_1\left(\mathbb{E}[UI(w)] + s\mathbb{E}[w_1]\right)$ *and* $K_2 = (1+r)(\mathbb{E}[w_2] - e_2)$. $\mathbb{E}[w_i]$ *are the expected value of wage offers in each sector* $i = 1,2$.

Proposition 6 shows that V^U is decreasing in effort costs and increasing in b, in the expected wage offers $\mathbb{E}[w_1]$ and $\mathbb{E}[w_2]$, in the unemployment benefit parameters s and $\mathbb{E}[UI]$, and in the job-finding probability ϕ. Additionally, V^U is increasing in ϕ_1 whenever $K_1/(r+q_1) > K_2/(r+q_2)$, that is, when the average formal job is more valuable than an average informal one.

---

[56] For notational simplicity, we write the profit functions in Equation 2 and Equation 4 assuming all workers exert effort, which is true in equilibrium as argued in Section D.2.2. More generally, the firm's effective labor force is $\mathbb{1}\{e(w) = e_i\}\ell_i(w)$, where $e(w) = e$ if and only if, w satisfies the NSCs in Equations D.4 and D.7.

D.3 Steady-State Equilibrium

D.3.1 Labor Market Clearing and Equilibrium Unemployment

Let M_1 and M_2 be the endogenous measures of workers in the formal and informal sectors, respectively, and U the measure of those in unemployment, such that $M_1 + M_2 + U = M$, where M is the measure of workers in the labor force. Note that, by imposing the labor supply curves in the firms' problem, labor markets automatically clear. Hence, we can look at the aggregate labor demand in each sector $i = 1,2$, which will match aggregate labor supply:

$$M_i = \mu_i \cdot \int_{\underline{w}_i}^{\infty} \ell_i(w)\, dF_i(w) \tag{D.12}$$

with $F_i(w)$ defined in Proposition 3. Unemployment is calculated as the residual $U = M - M_1 - M_2$, with the sector shares being given by $m_i = M_i/M$, $i = 1,2$, and $u = U/M$. A direct measure of worker (in)formality is $\tilde{m}_i = M_i/(M_1 + M_2)$ for $i = 1,2$.

D.3.2 Job-Finding Rates

As mentioned in Section 5.2, the job-finding rates ϕ, ϕ_1 and ϕ_2 are endogenous objects, although treated as exogenous by firms and workers. To find them, we rely on steady-state equilibrium conditions. We illustrate the worker flows in and out of each sector in Figure D.1.

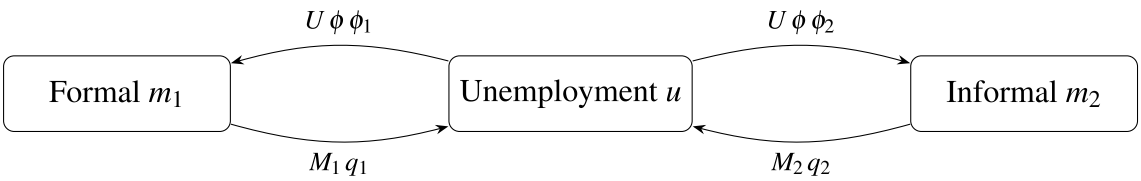


Figure D.1: Steady-State Flows Between Sectors

By Equation D.8, the flow into the formal sector is $U \cdot \phi \cdot \phi_1$, while the flow into the informal one is $U \cdot \phi \cdot \phi_2$. Since we do not have shirking in equilibrium, job separations only occur for exogenous reasons, so the flow into unemployment is $M_1 \cdot q_1$ from the formal sector and is $M_2 \cdot q_2$ from the informal sector. Since there is no on-the-job-search, these are the only entry and exit flows. Therefore, the following flow relationships must be true in steady-state:

$$\begin{cases} M_1 \cdot q_1 = U \cdot \phi \cdot \phi_1 \\ M_2 \cdot q_2 = U \cdot \phi \cdot \phi_2, \end{cases} \tag{D.13}$$

where the unemployment steady-state condition $M_1 \cdot q_1 + M_2 \cdot q_2 = U \cdot \phi$ is the sum of the two equations.[57] Intuitively, ϕ, ϕ_1 and ϕ_2 rationalize the equilibrium values of m_1, m_2 and u, given the exogenous separation rates q_1 and q_2.

---

[57] If one makes $M_1 = 0$ or $M_2 = 0$ – so that the model only has one sector –, Equation D.13 is the same as Shapiro and Stiglitz (1984, Equation 10).

D.3.3 Equilibrium

We adopt a Nash equilibrium concept where firms optimally choose sectors and set wages, workers make optimal effort choices and steady-state conditions are met.[58]

Definition 1 (Equilibrium)**.** *A steady-state equilibrium is defined as labor supply curves $\ell_i(w)$, distributions of wage offers $F_i(w)$, entry thresholds $\underline{\theta}_i$, measures of active firms μ_i, sector sizes M_i, U and job-finding rates ϕ, ϕ_i such that, in each sector $i = 1, 2$,*

(i) ***No Shirking Conditions****: The NSCs in Equations D.4 and D.7 hold with equality in both sectors, such that firm-level labor supply curves are given by Equation D.10.*

(ii) ***Wage-Setting****: Active firms choose wages to maximize profits subject to the firm-level labor supply curves implied by the NSCs (Propositions 1 and 2).*

(iii) ***Entry****: Entrants choose sectors to maximize expected profits (Equation 11), there is free-entry (Equation 12), and sector sizes are constant (Equation 13).*

(iv) ***Labor Market Clearing and Workforce Sizes****: Workforce sizes M_1, M_2 and U and job-finding rates ϕ, ϕ_1 and ϕ_2 are consistent with Equations D.12 and D.13.*

Timing. At the start of every period, active firms pay their workers, who choose whether to shirk or to exert effort, and production takes place. All transitions happen at the end of the period. First, firms exogenously exit and their labor force is inherited by an equivalent successful entrant with the same productivity, as the distribution of successful entrants is the same as that of exiting incumbents by the steady-state assumption.[59] Second, workers are exogenously fired and are replaced by unemployed workers; recall that there is no shirking in equilibrium. Then, the period ends. This timing ensures all successful entrants produce for at least one period, which is consistent with the form of their value functions.

---

[58] Albrecht and Vroman (1998, Footnote 3) note that Shapiro and Stiglitz (1984, p. 437) state their equilibrium as a Nash equilibrium in a wage-setting game, but solve it with "quasi-Walrasian" methods.

[59] In a single-sector model which combines Shapiro and Stiglitz (1984) efficiency wages with Melitz (2003) entry in the context of international trade, Davis and Harrigan (2011) treat $q_i = \delta_i$. Note that unemployment through firm exit or by layoffs from an active firm are observationally equivalent in our worker-level survey data.

D.4 Computing the Value of Unemployment

This appendix details how we numerically calculate the level and changes in V^U to take into account labor supply changes in our minimum wage counterfactuals.

Equation D.8 shows that a minimum wage increase can cause several changes in V^U: it may increase unemployment by driving formal firms out of the market, decreasing ϕ and V^U. It may also cause formal firms to switch into informality where businesses are smaller, decreasing ϕ_1 and increasing ϕ_2: if formal jobs have higher values than informal ones, this also shrinks V^U. On the other hand, a higher $w_{\min}$ increases the expected value of wage offers in both sectors, increasing V^U. As $\ell_i(\cdot)$ is decreasing in V^U and V^U does not depend on any w, we assume that:

$$\Lambda_i = \frac{\tilde{\Lambda}_i}{V^U} \tag{D.14}$$

To find V^U, we first need to calculate the value of formal benefits and the job-finding rates. As detailed in Appendix C, the severance rate s is taken from the implied values in Haanwinckel and Soares (2021), and τ_w from Ulyssea (2010), while the expected value of unemployment insurance $\mathbb{E}[UI]$ is endogenously determined using the institutional values of the last quarter of 2003 and the model's equilibrium formal wage offer distribution. The monthly worker discount rate $r \approx 1\%$ is chosen to match Brazil's average real risk-free rate (Selic) in 2003.

The exogenous job separation probabilities q_1 and q_2 are estimated from the PME panel.[60] To calculate ϕ, ϕ_1 and ϕ_2, we calibrate M so that the unemployment rate u in the model matches its empirical counterpart. With M in hand, we can calculate m_1, m_2 and u and use Equation D.13 to find ϕ, ϕ_1 and ϕ_2. The expected wage values can be found with the results in Proposition 6.

Finally, we need the unemployment flow value b and the cost of effort in each sector e_1 and e_2. To find them, we impose $e_1 = e_2 = e$ and use the following equilibrium conditions:

$$\begin{cases} V_2(\underline{w}_2) = V^U \\ \lim_{\ell \to \ell_2(\underline{w}_2)} D_2(\ell) = 1 \end{cases} \tag{D.15}$$

The first condition comes from profit maximizing behavior and the fact that all unemployment is involuntary and is similar to Meghir, Narita, and Robin (2015): the informal firm that offers the lowest wage must make the worker indifferent between accepting the job and unemployment. The second condition imposes that this same firm – which is the smallest in the economy – can almost perfectly monitor its workers. These two conditions allow us to numerically solve for (b, e) given all other parameters.

We can now calculate V^U in the baseline estimated economy and find $\tilde{\Lambda}_i$ using Equa-

---

[60] We use monthly PME data from April to December of 2003, a period during which the minimum wage was constant. We follow Meghir, Narita, and Robin (2015) and consider only the first four interviews and additionally filter for observations in consecutive months. We impose the same sample criteria as in the PNAD data and estimate monthly rates of $q_1 = 1.1\%$ and $q_2 = 5.2\%$, in line with Meghir, Narita, and Robin (2015).

tion D.14. Then, on top of "plain" counterfactuals where we just re-solve the model with the higher minimum wage, we also do simulations considering changes in V^U using a fixed point algorithm: for each guess of V^U, we calculate Λ_i using Equation D.14, solve the model and find the resulting V^U with Equation D.11, iterating until convergence.

Table D.1 shows the values of the objects estimated in this section. Informal jobs are twice as visible, but have a lower average wage offer. To accommodate the existence of very low informal wages, we estimate a small cost of effort e and a negative b, which is common in many models (Hornstein, Krusell, and Violante 2011; Meghir, Narita, and Robin 2015; Haanwinckel and Soares 2021), especially since unemployment insurance is a separate parameter in the model. Figure H.3 shows the implied monitoring probabilities $D_2(\ell)$.

Table D.1: Parameter Estimates – Changes in Market Conditions

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
m_1	0.53	ϕ	0.07	q_1	0.01	$E[w_1]$	463.15
m_2	0.22	ϕ_1	0.34	q_2	0.05	$E[w_2]$	284.16
u	0.25	ϕ_2	0.66	$E[UI]$	1223.04	e	15.17
r	0.01	τ_w	0.10	s	0.79	b	-621.82

Notes: Parameters are in monthly terms and R$ 2003 values, when applicable. Expected values are with respect to the distribution of offers F_i. *Source*: model simulations.

D.5 Proofs

Proof of Proposition 1. We start with the informal sector. Fix p. The FOC of Equation 2 is:

$$\frac{d\pi_2}{dw} = 0 = \frac{d\ell_2(w)}{dw} \cdot [p - w] - \ell_2(w) - \frac{dC(\ell_2(w))}{dw}$$

We assume that $C(\ell_2)$ does not depend directly on the wage, such that its total derivative with respect to w can be written as:

$$\frac{dC(\ell_2(w))}{dw} = \underbrace{\frac{dC(\ell_2(w))}{d\ell_2(w)}}_{:=C'(\ell_2(w))} \cdot \underbrace{\frac{d\ell_2(w)}{dw}}_{:=\ell_2'(w)}$$

Substituting in the FOC and dividing by $\ell_2'(w) > 0$ gives:

$$\frac{\ell_2(w)}{\ell_2'(w)} = (p - w) - C'(\ell_2(w))$$

The derivative of the RHS with respect to w is $-1 - C''(\ell_2(w)) \cdot \ell_2'(w)$. Since $C(\cdot)$ is convex by assumption, $C'' > 0$ and the RHS is thus strictly decreasing. For a unique solution, we need the LHS to be increasing, which is assured by the log-concavity of $\ell_2(w)$ (Kline 2025). To see it, note that:

$$\frac{d\log \ell_2(w)}{dw} = \frac{\ell_2'(w)}{\ell_2(w)}$$

Since $\ell_2(w)$ is log-concave, $d^2 \log \ell_2(w)/dw^2 < 0$, so ℓ_2'/ℓ_2 is strictly decreasing, which implies ℓ_2/ℓ_2' is strictly increasing. By the Intermediate Value Theorem, the FOC has a unique solution. To see it is a maximum, shift ℓ_2/ℓ_2' back to the RHS and note that all terms are strictly decreasing in w, which implies $\pi_2^A(w;p)$ is strictly concave in w.

For the formal sector, similar calculations yield the FOC:

$$(1+\tau) \cdot \frac{\ell_1(w)}{\ell_1'(w)} = p - (1+\tau) \cdot w$$

A similar argument to the informal case reveals that there is a unique wage that solves the FOC, which completes the first part of the proposition. As the profit value function is strictly concave – and so decreasing in w for wages greater than the interior solution –, enforcing the minimum wage constraint gives the second part of the proposition.

We now turn to the equilibrium wage offer distribution of Type-A firms in each sector. We again start with the informal sector. Since $w_2^*(p)$ is always an interior solution, $F_2^A(w)$ is a transformation of $\Gamma_2(p)$: $F_2^A(w_2^*(p)) = \Gamma_2(p)$ for $p \in \text{supp}(\Gamma_2)$.

In the formal sector, we have to account for the minimum wage $w_{\min}$. To characterize

$F_1^A(w)$, let $p_1^{w_{\min}}$ be the productivity of the "last" formal Type-A firm constrained by the minimum wage: $p_1^{w_{\min}} = \sup\{p : w_1^*(p) = w_{\min}\}$. For expositional purposes, we assume $p_1^{w_{\min}} > \underline{p}_1$, so $p_1^{w_{\min}}$ is in the support of the continuous productivity distribution $\Gamma_1(p)$. Then, note that all formal firms with $p \in [\underline{p}_1, p_1^{w_{\min}}]$ post the minimum wage and firms with $p > p_1^{w_{\min}}$ have an interior solution. Therefore, for $p \in \text{supp}(\Gamma_i)$, $i = 1,2$:

$$F_1^A(w_1^*(p)) = \begin{cases} \Gamma_1\left(p_1^{w_{\min}}\right), & p \leq p_1^{w_{\min}} \\ \Gamma_1(p), & p > p_1^{w_{\min}} \end{cases} \quad \text{and} \quad F_2^A(w_2^*(p)) = \Gamma_2(p)$$

Since $\Gamma_i(p)$ is absolutely continuous and in both sectors $i = 1,2$, the wage offer distributions have a connected support and, except for the mass point at $w_{\min}$ in the formal sector, are also absolutely continuous. ■

Proof of Proposition 2. Fix $i \in \{1,2\}$ and $p \in \text{supp}(\Gamma_i)$. By Proposition 1, let $w_i^*(p)$ be the (unique) optimal wage chosen by the Type-A firm, such that $\pi_i(w_i^*(p);p) > \pi_i(w;p)$ for all $w \neq w_i^*(p)$ that satisfy the sector-specific restrictions of the firm. The assumption that firms of different types are identical up to p and i is useful here because we can use the Type-A solution as a benchmark for the Type-B problem.

Consider now the problem of a Type-B firm. To solve their problem, it will be useful to define the difference in profits:

$$\Delta_i(p) = (1-\kappa_i)\pi_i^A(w_i^*(p);\, p) - \pi_i^A(w_{\text{min}};\, p), \tag{D.16}$$

Since $\ell_i(w)$ is continuous in w by Proposition 5 and $C(\ell_2(w))$ is continuous in ℓ_2 by assumption, $\pi_i^A(w;p)$ is continuous in both w and p, so $\Delta_i(p)$ is continuous in p for both $i = 1,2$. By Equation 4, κ_i does not enter the first-order condition. Then, the problem of the Type-B firm becomes a threshold evaluation: it chooses to post $w_i^*(p)$ – the optimal wage of the "equivalent" Type-A firm – if, and only if,

$$\Delta_i(p) = (1-\kappa_i)\pi_i^A(w_i^*(p);p) - \pi_i^A(w_{\text{min}};p) \geq 0$$

and posts w_{min} otherwise. We adopt the convention that the Type-B firm posts $w_i^*(p)$ when it is indifferent between it and w_{min}, i.e., when $\Delta_i(p) = 0$. Note that:

$$\begin{aligned}
\frac{d\Delta_i(p)}{dp} &= \frac{d\,(1-\kappa_i)\pi_i^A(w_i^*(p);p)}{dp} - \frac{d\,\pi_i^A(w_{\text{min}};p)}{dp} \\
\text{(Envelope, Fixed } w\text{)} &= \left.\frac{\partial\,(1-\kappa_i)\pi_i^A(w;p)}{\partial p}\right|_{w=w_i^*(p)} - \frac{\partial\,\pi_i^A(w_{\text{min}};p)}{\partial p} \\
\text{(Profits)} &= (1-\kappa_i)\ell_i(w_i^*(p)) - \ell_i(w_{\text{min}}) \\
&\geq 0 \iff p \geq \mathring{p}_i
\end{aligned}$$

where $\mathring{p}_i$ is defined by the equality $(1-\kappa_i)\ell_i(w_i^*(\mathring{p}_i)) = \ell_i(w_{\text{min}})$.

The Intermediate Value Theorem (IVT), $\text{supp}(\Gamma_i) = [\underline{p}_i, \infty)$, $\ell_i'(w) > 0$, and $w_i^{*'}(p) > 0$ (Proposition 1) imply a unique $\mathring{p}_i \in \text{supp}(\Gamma_i)$.[61] $\Delta_i(p)$ then has a U-shape, being strictly decreasing for $p < \mathring{p}_i$ and strictly increasing for $p > \mathring{p}_i$.

Take $i = 1$ and assume that $p_1^{w_{\text{min}}} > \underline{p}_1$, so $w_1^*(\underline{p}_1) = w_{\text{min}}$. As active firms must have positive profits, $\Delta_1(\underline{p}_1) = -\kappa_1 \cdot \pi_1^A(w_{\text{min}};\underline{p}_1) < 0$. Since $\mathring{p}_1$ is finite and $\Delta_1'(p) > 0$ for $p > \mathring{p}_1$, the IVT implies there exists a unique threshold $p_1^* > \mathring{p}_1$ such that $\Delta_1(p_1^*) = 0$, and so Type-B firms with $p \geq p_1^*$ have $\Delta_1(p) \geq 0$ and prefer to post $w_1^*(p)$ instead of w_{min}. This means that more productive Type-B firms prefer to post the optimal contract $w_1^*(p)$ and face the cost κ_1. For

---

[61] Technically, $w_1^{*'}(p) > 0 \iff p > p_1^{w_{\text{min}}}$ (i.e., for firms unconstrained by the minimum wage), but $\text{supp}(\Gamma_1) = [\underline{p}_1, \infty)$ guarantees the existence of a region where $w_1^{*'}(p) > 0$.

$p < p_1^*$, it is optimal for the Type-B firm to post $w_{\min}$.

Note that $p_1^* > p_1^{w_{\min}}$ if $\kappa_1 > 0$. If $w_1^*(p) = w_{\min}$, the Type-B firm posts $w_{\min}$. By continuity of $\Delta_1(p)$, $w_1^*(p)$ (Proposition 1) and of the profit functions, there exists $\bar{\varepsilon} > 0$ such that, for all $0 < \varepsilon < \bar{\varepsilon}$:

$$(1-\kappa_1)\pi_1^A(w_1^*(p_1^{w_{\min}}+\varepsilon); p_1^{w_{\min}}+\varepsilon) < \pi_1^A(w_{\min}; p_1^{w_{\min}}+\varepsilon), \tag{D.17}$$

and so the formal Type-B firm with productivity "just above" $p_1^{w_{\min}}$ also prefers to post $w_{\min}$ (i.e., $\Delta_1(p_1^{w_{\min}}+\varepsilon) < 0$ for all $0 < \varepsilon < \bar{\varepsilon}$). The result on the wage offer CDF of formal Type-B firms is immediate given the above; one can also substitute $p_1^{w_{\min}}$ by $\underline{p}_1$ without loss.

Suppose now that $i = 2$. The sign of $\Delta_2(\underline{p}_2)$ is ambiguous, as informal firms are not constrained by the minimum wage. However, note that, by continuity of π_2 and $w_2^*(p)$, there exists a $p \in \text{supp}(\Gamma_2)$ where $\Delta_2(p) < 0$ (similar to Equation D.17).

If $\Delta_2(\underline{p}_2) > 0$, then there will exist a unique threshold $p_{2*} < \mathring{p}_2$ such that $\Delta_2(p_{2*}) = 0$ (since $\Delta_2'(p) < 0$ for $p < \mathring{p}_2$).

If $\Delta_2(\underline{p}_2) < 0$, then we take $p_{2*} = \underline{p}_2$. Similarly to the formal case, there also exists a unique threshold $p_2^* > \mathring{p}_2 > p_{2*}$ such that $\Delta_2(p_2^*) = 0$, and so firms with $p \geq p_2^*$ post $w_2^*(p)$.

To get the CDF of informal Type-B firms, note that, by the Law of Total Probability,

$$\begin{aligned} F_2^B(w \mid p_{2*} < p < p_2^*) = {} & \mathbb{P}(p \geq \mathring{p}_2 \mid p_{2*} < p < p_2^*) \cdot F_2^B(w \mid p_{2*} < p < p_2^*, p \geq \mathring{p}_2) \\ & + \mathbb{P}(p < \mathring{p}_2 \mid p_{2*} < p < p_2^*) \cdot F_2^B(w \mid p_{2*} < p < p_2^*, p < \mathring{p}_2) \end{aligned}$$

Both CDFs are step functions by the above results, as informal firms that satisfy both conditionals post $w_{\min}$. Hence,

$$F_2^B(w \mid p_{2*} < p < p_2^*, p \geq \mathring{p}_2) = F_2^B(w \mid p_{2*} < p < p_2^*, p < \mathring{p}_2) = \begin{cases} 0, w < w_{\min} \\ 1, w \geq w_{\min} \end{cases}$$

Since $\mathbb{P}(p \geq \mathring{p}_2 \mid p_{2*} < p < p_2^*) + \mathbb{P}(p < \mathring{p}_2 \mid p_{2*} < p < p_2^*) = 1$, $F_2^B(w \mid p_{2*} < p < p_2^*)$ is the same step function. ■

Proof of Proposition 3. The claim on only one mass point follows directly from Propositions 1 and 2. We now prove connected support. Fix $i \in \{1,2\}$ and use Proposition 2 and the Law of Total Probability to write F_1^B as:

$$\begin{aligned}
F_i^B(w_i^*(p)) = {} & \mathbb{P}(p \leq p_{i*}) \cdot F_i^B(w_i^*(p) \mid p \leq p_{i*}) \\
& + \mathbb{P}(p_{i*} < p < p_i^*) \cdot F_i^B(w_i^*(p) \mid p_{i*} < p < p_i^*) \\
& + \mathbb{P}(p \geq p_i^*) \cdot F_i^B(w_i^*(p) \mid p \geq p_i^*) \\
= {} & \mathbb{P}(p \leq p_{i*}) \cdot F_i^A(w_i^*(p) \mid p \leq p_{i*}) \\
& + \mathbb{P}(p_{i*} < p < p_i^*) \cdot \mathbb{1}\{w_i^*(p) \geq w_{\min}\} \\
& + \mathbb{P}(p \geq p_i^*) \cdot F_i^A(w_i^*(p) \mid p \geq p_i^*),
\end{aligned}$$

Using this result, we can add and subtract $(1-\sigma_i) \cdot \mathbb{P}(p_{i*} < p < p_i^*) \cdot F_i^A(w_i^*(p) \mid p_{i*} < p < p_i^*)$ to Equation 5 to get:

$$\begin{aligned}
F_i(w_i^*(p)) &= \sigma_i F_i^A(w_i^*(p)) + (1-\sigma_i) F_i^B(w_i^*(p)) \\
&= F_i^A(w_i^*(p)) + (1-\sigma_i) \cdot \mathbb{P}(p_{i*} < p < p_i^*) \cdot \left[\mathbb{1}\{w_i^*(p) \geq w_{\min}\} - F_i^A(w_i^*(p) \mid p_{i*} < p < p_i^*)\right],
\end{aligned}$$

which gives Equation 6. Then, if $\sigma_i = 0$, $F_i(w)$ is flat for $w \in (w_i^*(p_{i*}), w_i^*(p_i^*)) \setminus \{w_{\min}\}$. As $F_i^A(w)$ has a connected support (Proposition 1), $\sigma_i > 0$ ensures $F_i(w)$ will also have one.

One can also write the wage offer CDFs as a function of the productivity distribution of active firms using Proposition 1:

$$F_1(w_1^*(p)) = \begin{cases} \Gamma_1(p_1^{w_{\min}}) + (1-\sigma_1) \cdot (\Gamma_1(p_1^*) - \Gamma_1(p_1^{w_{\min}})) \cdot [1-0], & p \leq p_1^{w_{\min}} \\ \Gamma_1(p) + (1-\sigma_1) \cdot (\Gamma_1(p_1^*) - \Gamma_1(p_1^{w_{\min}})) \cdot [1 - \Gamma_1(p \mid p_1^{w_{\min}} < p < p_1^*)], & p_1^{w_{\min}} < p < p_1^* \\ \Gamma_1(p), & p \geq p_1^* \end{cases}$$

$$F_2(w_2^*(p)) = \Gamma_2(p) + (1-\sigma_2) \cdot (\Gamma_2(p_2^*) - \Gamma_2(p_{2*})) \cdot [\mathbb{1}\{w_2^*(p) \geq w_{\min}\} - \Gamma_2(p \mid p_{2*} < p < p_2^*)],$$

where $p \leq p_1^{w_{\min}}$ is equivalent to $w = w_{\min}$, which justifies $F_1^A\left(w_{\min} \mid p_1^{w_{\min}} < p < p_1^*\right) = 0$ in the first line, and $(p_1^{w_{\min}} < p < p_1^*)$ to $(w_{\min} < w < w_1^*(p_1^*))$.

Recall that by Proposition 1, the mapping $F_i^A(w_i^*(p)) = \Gamma_i(p)$ holds for all $p \in \operatorname{supp}(\Gamma_1) \setminus \{p_1^{w_{\min}}\}$ and all $p \in \operatorname{supp}(\Gamma_2)$ — that is, everywhere except at the formal sector mass point $w_1^*(p) = w_{\min}$. Differentiating with respect to p and applying the chain rule then yields $f_i^A(w_i^*(p))\, w_i^{*\prime}(p) = \gamma_i(p)$, and differentiating Equation 6 with respect to w gives Equation 7 for all $w \neq w_{\min}$. ■

Proof of Proposition 4. We prove the result for the formal sector, as making $\tau_w = UI = s = 0$ and changing subscripts yields the informal sector equation.

We can isolate $V_1^N(w)$ and $V_1^S(w)$ in Equations D.2 and D.3 to get:

$$V_1^N(w) = \frac{(1-\tau_w)w - e_1 + \frac{1}{1+r}q_1(V^U + UI + s\cdot w)}{1 - \frac{1}{1+r}(1-q_1)}$$

$$V_1^S(w) = \frac{(1-\tau_w)w + \frac{1}{1+r}V^U[q_1 + (1-q_1)D_1(\ell)] + \frac{1}{1+r}q_1(UI + s\cdot w)}{1 - \frac{1}{1+r}(1-q_1)(1-D_1(\ell))}$$

We can multiply numerators and denominators by $1+r$ to get:

$$V_1^N(w) = \frac{(1+r)(1-\tau_w)w - (1+r)e_1 + q_1(V^U + UI + s\cdot w)}{\underbrace{r+q_1}_{=A_1}}$$

$$V_1^S(w) = \frac{(1+r)(1-\tau_w)w + [q_1 + \overbrace{(1-q_1)D_1(\ell)}^{=B_1}]V^U + q_1(UI + s\cdot w)}{\underbrace{r+q_1+(1-q_1)D_1(\ell)}_{=A_1+B_1}}$$

Then, $V_1^N(w) = V_1^S(w)$ for all w implies:

$$\begin{aligned}
&\left\{(1+r)(1-\tau_w)w - (1+r)e_1 + q_1(V^U + UI + s\cdot w)\right\}(A_1+B_1) \\
&= \left\{(1+r)(1-\tau_w)w + (q_1+B_1)V^U + q_1(UI + s\cdot w)\right\}A_1 \\
&\therefore [(1+r)(1-\tau_w) + q_1 s]\,wB_1 = (1+r)e_1(A_1+B_1) - UI\,q_1B_1 + V^U\overbrace{(A_1B_1 - q_1B_1)}^{=rB_1}
\end{aligned}$$

Substituting back the expressions for A_1 and B_1 and isolating $D_1(\ell)$ gives the implied monitoring technologies. Equality holds as w will be optimally chosen by firms so that the NSCs hold exactly. ■

Proof of Proposition 5. The expressions of $\ell_1(w)$ and $\ell_2(w)$ follow directly from the monitoring probabilities $D_1(\ell)$ and $D_2(\ell)$ in Proposition 4 and the fact that they are invertible (Assumption 1). Twice continuous differentiability also follows directly from Assumption 1.

To see that the curves are strictly increasing, fix $i \in \{1,2\}$ and let $d_i(x) = D_i^{-1}(x)$. The chain rule gives:

$$\ell_i'(w) = \frac{d\,d_i(g_i(w))}{d\,g_i(w)} \cdot \frac{d\,g_i(w)}{d\,w} = d_i'(g_i(w)) \cdot g_i'(w), \quad \text{where}$$

$$\begin{aligned} g_i(w) &= \frac{a_i}{b_i + c_i \cdot w} \\ a_i &= e_i(r+q_i) \\ b_1 &= -(1-q_1)\left(e_1 + \frac{r}{1+r}V^U - \frac{q_1}{1+r}U\right) \qquad & b_2 &= -(1-q_2)\left(e_2 + \frac{r}{1+r}V^U\right) \\ c_1 &= (1-q_1)\left((1-\tau_w) + \frac{q_1 \cdot s}{1+r}\right) & c_2 &= (1-q_2) \end{aligned}$$

where $a_i > 0$ and $b_i + c_i \cdot w > 0$ for $i = 1,2$ and all relevant w (otherwise, $D_i(\ell)$ would not be a valid probability). Note also that $d_i(g_i(w)) > 0$ for all w, since labor supply must be positive.

Since $D_i'(x) < 0$ by Assumption 1, $d_i'(x) < 0$ for $i = 1,2$. Noting that V^U in Equation D.8 is not a function of the current wage w (although it is a function of the distribution of wage offers), we have that:

$$g_i'(w) = \frac{-a_i \cdot c_i}{(b_i + c_i \cdot w)^2}$$

Since $q_i \in (0,1)$, $\tau_w \in [0,1]$, $e_i > 0$ for $i = 1,2$, and $r, s \geq 0$, we have $g_i'(w) < 0$ for $i = 1,2$. Therefore, $\ell_1'(w) > 0$ and $\ell_2'(w) > 0$.

To assess log-concavity, recall that positive, twice differentiable functions are log-concave, if, and only if, $\ell_i''(w) \cdot \ell_i(w) \leq (\ell_i'(w))^2$. The chain rule gives:

$$\ell_i''(w) = d_i''(g_i(w)) \cdot g_i'(w)^2 + d_i'(g_i(w)) \cdot g_i''(w)$$

For notational simplicity, we omit sector subscripts from now on. Note that:

$$g''(w) = \frac{2ac^2}{(b + c \cdot w)^3} > 0$$

Substituting the expressions for $\ell(w)$, $\ell'(w)$ and $\ell''(w)$ in the condition for log-concavity:

$$\begin{aligned}
\Big[d''(g(w))\cdot(g'(w))^2+d'(g(w))\cdot g''(w)\Big]d(g(w)) &\leq \Big[d'(g(w))\cdot g'(w)\Big]^2 \\
(\cdot/(d\cdot g'^2)>0)\; d''(g(w))+\frac{d'(g(w))\cdot g''(w)}{g'(w)^2} &\leq \frac{d'(g(w))^2}{d(g(w))} \\
d''(g(w)) &\leq \underbrace{\frac{d'(g(w))^2}{d(g(w))}}_{+}-\underbrace{\frac{d'(g(w))\cdot g''(w)}{g'(w)^2}}_{-},
\end{aligned}$$

where the signs come from the fact that $d>0$, $d'<0$ and $g''>0$ (note that the negative sign on the second term does not include the $-$ sign upfront). By Assumption 1, $d=D^{-1}$ is log-concave, which implies:

$$\begin{aligned}
d''(g(w)) &\leq \frac{d'(g(w))^2}{d(g(w))} && \text{(Assumption 1)} \\
&< \frac{d'(g(w))^2}{d(g(w))}-\frac{d'(g(w))\cdot g''(w)}{g'(w)^2}
\end{aligned}$$

Therefore, log-concavity of d is a sufficient, but not necessary, condition for log-concavity of ℓ. ■

Proof of Proposition 6. We first derive the expected values of exerting effort. Isolating $V_1^N(w)$ in Equation D.2 and multiplying numerator and denominator by $(1+r)$:

$$V_1^N(w) = \frac{(1+r)(1-\tau_w)w-(1+r)e_1+q_1\left(V^U+UI+s\cdot w\right)}{r+q_1}$$

Since V^U is not a function of any particular wage, taking expectations with respect to $F_1(w)$ gives:

$$\mathbb{E}\left[V_1^N(w)\right] = \frac{(1+r)\left((1-\tau_w)\mathbb{E}[w_1]-e_1\right)+q_1\left(\mathbb{E}[UI(w)]+s\cdot\mathbb{E}[w_1]\right)+q_1V^U}{r+q_1} = \frac{K_1+q_1V^U}{r+q_1}$$

where $K_1 = (1+r)\left((1-\tau_w)\mathbb{E}[w_1]-e_1\right)+q_1\left(\mathbb{E}[UI(w)]+s\cdot\mathbb{E}[w_1]\right)$. Similarly, isolating $V_2^N(w)$ in Equation D.5:

$$V_2^N(w) = \frac{(1+r)(w-e_2)+q_2V^U}{r+q_2}$$

Taking expectations with respect to $F_2(w)$:

$$\mathbb{E}\left[V_2^N(w)\right] = \frac{(1+r)\left(\mathbb{E}[w_2]-e_2\right)+q_2V^U}{r+q_2} = \frac{K_2+q_2V^U}{r+q_2}$$

where $K_2 = (1+r)\left(\mathbb{E}[w_2]-e_2\right)$. Substituting in Equation D.8 and multiplying both sides by $(1+r)$ gives:

$$(1+r)V^U = (1+r)b+\phi\left(\phi_1\frac{K_1+q_1V^U}{r+q_1}+\phi_2\frac{K_2+q_2V^U}{r+q_2}\right)+(1-\phi)V^U$$

We can collect terms in V^U on the left-hand side to get:

$$\underbrace{\left[r+\phi-\phi\left(\frac{\phi_1q_1}{r+q_1}+\frac{\phi_2q_2}{r+q_2}\right)\right]}_{>r>0}V^U = (1+r)b+\phi\left(\phi_1\frac{K_1}{r+q_1}+\phi_2\frac{K_2}{r+q_2}\right)$$

Dividing both sides by the coefficient on V^U yields the expression in the proposition. ■

E Models with Wage Distributions and Mass Points

This appendix discusses mechanisms commonly used to generate equilibrium wage dispersion – focusing on the channels that do so among homogeneous workers – and models that feature a mass point at the minimum wage in the formal sector.

Mechanisms for Wage Dispersion. Our model generates wage dispersion for similar workers due to a combination of firm heterogeneity and an efficiency wage structure that creates an upward-sloping labor supply curve, mapping firm productivity into earnings (Propositions 1 and 2). Firm heterogeneity and monopsony power have also been used by Dube, Manning, and Naidu (2018, 2025), Reyes (2024), and Parente, Brotherhood, and Iachan (2025). Other models with monopsony power and wage dispersion, although needing heterogeneous workers, include Lamadon, Mogstad, and Setzler (2022), Amodio, Medina, and Morlacco (2024), and Haanwinckel (2025b).

Other efficiency wage models with wage dispersion – although with only one sector – include Ramaswamy and Rowthorn (1991) and Davis and Harrigan (2011), which feature firm heterogeneity in the differences between shirking and non-shirking output or monitoring technology, and Albrecht and Vroman (1998), which has an absolutely continuous distribution generated through worker heterogeneity in the disutility of effort.

Other approaches that can generate equilibrium wage dispersion among homogeneous workers include bargaining with convex vacancy-posting costs and concave production functions (Acemoglu and Hawkins 2014; Haanwinckel and Soares 2021) and wage-posting with on-the-job search (Burdett and Mortensen 1998; Van den Berg and Ridder 1998; Bontemps, Robin, and Van den Berg 2000; Postel–Vinay and Robin 2002; Van den Berg 2003; Carrillo-Tudela 2009; Meghir, Narita, and Robin 2015; Piyapromdee 2018; Engbom and Moser 2022).

Models with Mass Points. We now discuss recent models that generate bunching in the wage distribution. In particular, we focus on those that can create a mixed distribution with a connected support. For earlier single-sector frameworks with wage dispersion that allow for mass points, see Ramaswamy and Rowthorn (1991) and Bhaskar and To (2003).

First, Dube, Manning, and Naidu (2018, 2025) and Reyes (2024) build models with monopsony power with left-digit worker bias and mis-optimization by employers. To account for the connected support observed empirically, they make a similar assumption to ours: only a share of firms mis-optimize (Type-B), while others do not (Type-A). Dube, Manning, and Naidu (2018, 2025) extend the theory and consider more complex distributions of mis-optimization costs.

Parente, Brotherhood, and Iachan (2025) build a discrete choice model with firm-specific amenities, incorporating heterogeneity in firm productivity and two sectors, but without wage-anchoring. As in Ulyssea (2018), firms choose their sector to maximize expected profits based only on a signal of their true productivity, which can change after entry. Their model is able to generate a spike in the formal sector since firms with adverse post-entry shocks that do not exit the market would optimally post a lower wage, but are constrained by the minimum wage.

In Nash bargaining models, the spike can be rationalized using matches with heterogeneous productivity – either through idiosyncratic shocks, heterogeneous firm productivity or workers that are heterogeneous in the value of their outside option. If the bargained wage of a match is below the minimum but the match still yields positive surplus at w_{min}, then the firm opts to pay the minimum instead of receiving nothing (Flinn 2006; Flinn and Mullins 2021; Haanwinckel and Soares 2021; Bosch et al. 2025). Haanwinckel (2025b) uses a novel task-based production function approach to model the formal sector in Brazil where firms have monopsony power in the labor market. He generates a spike at w_{min} through a mechanism somewhat similar to the ones in the above Nash bargaining models.

For on-the-job-search wage-posting frameworks, the arguments in Burdett and Mortensen (1998) and Bontemps, Robin, and Van den Berg (2000) rule out equilibrium wage distributions with mass points, even when firms are heterogeneous in productivity: if there were a mass point, firms could offer a marginally higher wage and have a discrete gain in labor force size and profits, so bunching cannot be profit-maximizing. However, there are some modifications that can be made in order to generate a mass point at the lower bound of the wage distribution in this class of models.

Carrillo-Tudela (2009) shows that bunching at the minimum wage can arise when firms are allowed to discriminate workers based on their current employment status, offering lower wages to those unemployed. In the formal sector model of Engbom and Moser (2022), firms perfectly observe workers' ability and on-the-job search efficiency. The spike exists because there are low-ability workers who do not search while employed, and hence earn the minimum wage if it is above their reservation wage (Diamond 1971). Guo (2024) shows that a spike at the minimum wage can arise if vacancies are posted sequentially and jobs cannot start until some time in the future.

Albrecht, Carrillo-Tudela, and Vroman (2018) include amenities in the model of Burdett and Mortensen (1998). They assume workers have heterogeneous preferences over these amenities and that they are private information, so that firms cannot price them in wages as they do in Hwang, Mortensen, and Reed (1998). They show that, if the distribution of amenities is sufficiently dispersed, then there exists an equilibrium where all firms post the same wage, and so their model generates a step-function distribution. Moreover, the wage at which the step occurs is pinned down purely by algebra – it is a function of the dispersion of the amenity distribution and the reservation wage – and so carries no institutional interpretation analogous to w_{min}.

With the exception of the single-wage equilibrium of Albrecht, Carrillo-Tudela, and Vroman (2018), all of the above mechanisms only generate a spike in the formal sector and assume full compliance with a binding minimum wage. In particular, they are not able to rationalize the existence of a mass point in the interior of the informal wage distribution or to explain why it occurs exactly at the minimum wage, which is inconsistent with the empirical evidence of compensating differentials (Meghir, Narita, and Robin 2015; Haanwinckel and Soares 2021; Finamor 2025).

In sum, we abstract from on-the-job search because it precludes equilibrium wage distributions with interior mass points. This comes at some cost. First, we cannot capture informality acting as an unemployment buffer (e.g., Ulyssea (2020) and Engbom et al. (2022)); following Haanwinckel and Soares (2021), however, one may argue that informality does act as such a buffer, but through an intermediate unemployment spell that is not observed in the data. Second, we abstract from job ladders and employer competition, which can generate minimum wage spillovers across the wage distribution (Engbom and Moser 2022).

F Comparison with Parente, Brotherhood, and Iachan (2025)

This appendix compares our work with Parente, Brotherhood, and Iachan (2025) (PBI henceforth) and explores possible explanations for the different results. We want to make clear from the start that this is *not* a criticism of their work, but rather a discussion of the factors that may be behind the different results.

Data. PBI analyze the period 1996–2012, while our study period starts in 2003. They calculate that the real minimum wage increase was 105%, while it is 60% in our study period. We use the same dataset as them (PNAD), as well as the same wage measure (habitual gross monthly earnings). While they focus on individuals aged 18–54, our sample includes individuals aged 18–70. We focus on urban, private-sector workers and exclude domestic workers, while they do not write about any activity restriction.

On top of these differences, the main distinction is that we restrict attention to individuals who are full-time workers, following much of the analysis in Derenoncourt et al. (2025).[62] PBI do not impose an hours restriction: indeed, comparing our Figure A.1a (in standard deviations) with their Figure 1 (in variances) shows that the levels of inequality are similar in the formal sector, but not in the informal sector. Their levels are higher than ours, consistent with the fact that part-time work is more common in the informal sector.

Figure F.1 confirms these suspicions. When not restricting hours worked, the level of wage inequality in the informal sector increases (Panel (a)). In Panel (b), we show the dispersions of log hourly wages. In both cases, we see inequality fall in the formal and informal sectors, unlike Figure 1 of PBI; we suspect the difference is due to the activities we consider. Figure F.2 shows the distribution of hours worked, highlighting that over three quarters of informal employees

---

[62] It is worth noting that Derenoncourt et al. (2025)'s results on employment and small formal–informal reallocation include both domestic and part-time workers.

work at least 40 hours a week in all years between 2003 and 2012.

Design-Based Analysis. PBI conduct two design-based (or reduced-form) analyses. The first relies on the "fraction affected" difference-in-differences design (Card 1992) with a continuous measure of treatment intensity at the state level – the pre-treatment share of formal workers earning the minimum wage (their Equation 2). They split the 27 Brazilian states into nine treatment groups of three states each according to the pre-treatment bite of the minimum wage.[63] They find that states where the minimum was most binding experienced a relative decrease in formal inequality, but increases in informality and in informal and overall wage inequality.

These relative results are concentrated in the comparison of the three most treated versus the three least treated states, as highlighted by Firpo and Portella (2025).[64] Haanwinckel (2025a) also points out that this design can suffer from functional form misspecifications and trends in the dispersion of latent wages – which can arise from skill-biased technological change, a feature present in their model. Finally, the pre-trends tests that usually lend credibility to difference-in-differences designs are contaminated in the Brazilian case, as the real minimum wage had adjustments during 1996–1999, albeit smaller than those of the following decade.

They also adapt the method of Giupponi et al. (2024) to the Brazilian case, which itself adapts the frequency distribution approach of Cengiz et al. (2019) to settings without regional variation in minimum wages. A potential limitation of this approach in the Brazilian context is that the household surveys that cover the informal sector in Brazil may not provide sufficient controls to accurately estimate the regional wage premiums – for instance, Giupponi et al. (2024) use a large-scale matched employer-employee dataset reported by firms.

Like them, we focus on results using the Census, which is estimated at the microregion level and is more consistent with Giupponi et al. (2024). Interestingly, some of their results differ from those of the previous section: for instance, they find significant and "moderate" effects on unemployment and no effect on formal inequality. However, both analyses point to an increase in informality and in overall wage inequality.

Model-Based Analysis. Our equilibrium model and PBI's share a range of common features. Both model the labor market as monopsonistic competition and parametrize the labor supply curve with a common wage elasticity. Additionally, both feature the entry mechanism based on incomplete information of Ulyssea (2018).

The distinguishing feature of our model is the existence of wage-anchoring, which generates spikes at the minimum wage and missing masses above it in both sectors. Indeed, their Figure 11 shows that their model cannot reproduce these empirical facts. We also motivate the

---

[63] Derenoncourt et al. (2025) employ a similar strategy, but at the industry-state level and without treatment groups. They run regressions separately by sector (formal, informal or self-employed), using as the treatment variable the pre-treatment share of workers with a wage no larger than the minimum in each sector. They do not analyze wage inequality, but find an effect on informality much lower than PBI's in the period 2000–2010.

[64] Interestingly, their results in Figure 4 are non-monotonic across treatment intensities.

upward-sloping labor supply curve through efficiency wages, which rationalizes involuntary unemployment, while PBI's model creates this curve through amenities and does not feature unemployment.[65] Another central difference is that our model is dynamic – firms enter and exit the market in every period, while PBI have a fixed exogenous mass of firms. PBI have a constant informality penalty that multiplies productivity, while ours is additive and varies with firm size, as in Meghir, Narita, and Robin (2015), Ulyssea (2018), and Haanwinckel and Soares (2021).

PBI's richer production technology, featuring decreasing returns to scale and worker heterogeneity, represents a dimension where their model is more flexible than ours. This heterogeneity may also contribute to the differences in inequality results: a higher minimum wage may make firms substitute towards high-skill labor, increasing their salaries and decreasing low-skill wages, potentially causing a rise in inequality.

Effects of the Minimum Wage on Informality. We think that the biggest differences between the model results are the estimated effects on informality: PBI find that the 105% increase in the real minimum wage would have caused the informal sector to double (a 40 p.p. increase in the informal share of workers), while we find modest informality effects in line with Haanwinckel and Soares (2021) and Derenoncourt et al. (2025). At the same time, we estimate a higher counterfactual formal minimum wage spike, suggesting that some firms prefer to stay constrained by $w_{\min}$ in the formal sector rather than switch to the informal sector.

We point to two main factors behind these differences. First, our model is dynamic, and the exogenous probability of exit enters firms' value functions. Using the parameter values of Ulyssea (2018), formal firms have an exit rate that is almost one third that of informal firms, which means that the gradient of $\Pi_1(\theta)$ is generally steeper than that of $\Pi_2(\theta)$. This is compounded by the fact that we have entry costs, and so the formal threshold is shifted to the right – where the value function is even steeper due to the convexity of $\pi_1(p)$. The absence of entry costs and dynamic considerations in PBI's framework may allow more firms to be attracted towards informality, whereas in our model these firms prefer to remain constrained by $w_{\min}$ in the formal sector.[66] Second, the way informality costs are modeled may play a role. By making it a function of labor force size, firms with average productivity may not prefer to "switch" to the informal sector, as the size distortions may be more pronounced. Instead, they prefer to be constrained by $w_{\min}$ in the formal sector.

We do not think that the absence of decreasing returns to scale in the production function plays a significant role in the differences in results, as firms near the formality threshold are the smallest in that sector. Furthermore, differences in the estimated degree of monopsony (measured by the elasticity η) are small, so are also unlikely to drive the results. Finally, our counterfactuals

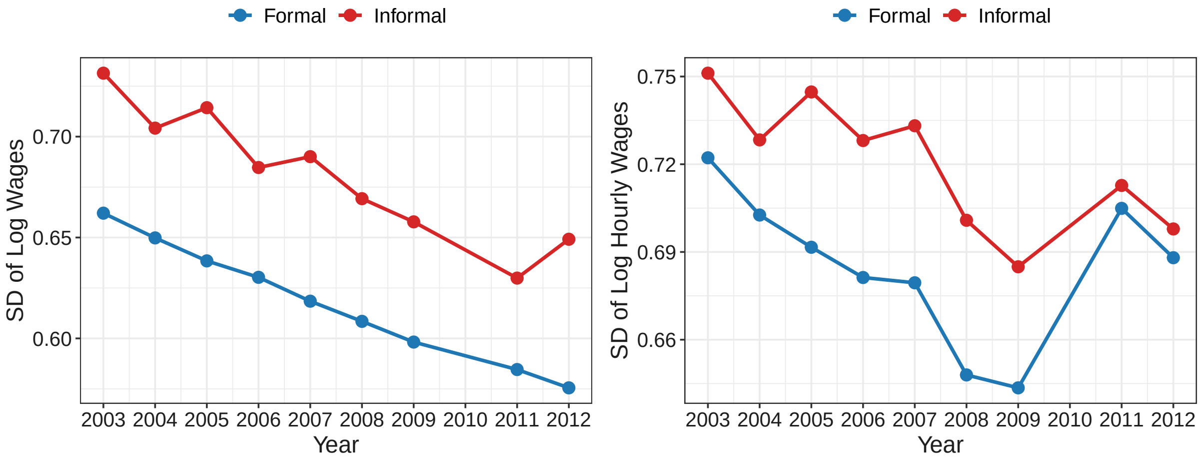
---

[65] PBI provide an extension with voluntary unemployment in their Appendix F.

[66] It is worth noting that the entry decision in our model is made by entrant firms and occurs every period; once made, it is final (Ulyssea 2018). In PBI, the mass of firms is fixed, so they must allow firms to change their compliance decision following a minimum wage change (Haanwinckel and Soares 2021). Without entry costs, all firms enter either the formal or informal sector, while entry costs create a mass of inactive entrants.

suggest that the differences are not due to the existence of wage-anchoring.

One may also think that the difference is due to the degree of the minimum wage increase considered, as PBI study a real variation that is almost double that of our study period. To shed light on this issue, we simulate a 105% increase in the minimum wage using the estimated structure for 2003, which is different from that of 1996. This exercise reveals that informality would have increased by 12 p.p. without changes in market conditions. This suggests that the starting point of the economy matters: as noted by Haanwinckel and Soares (2021), the estimated effects of the minimum wage are sensitive to initial economic conditions. The economy in 1996 may have had a structure less suited to accommodate such an increase – for instance, weaker enforcement or a different firm productivity distribution – while the one in 2003 could better absorb it. Indeed, PBI's findings on the complementarity between enforcement policies and the minimum wage support this interpretation.



(a) Wage Dispersion – All Hours

(b) Hourly Wage Dispersion – All Hours

Figure F.1: Wage Inequality Trends in Brazil (2003–2012) – No Hours Restrictions

Notes: Calculations take into account survey weights and exclude self-employed and public, agriculture and domestic sectors, as well as the bottom and top 1% of wage earners in each sector.
Source: PNAD 2003–2012. Use this to return to Section 7.1.

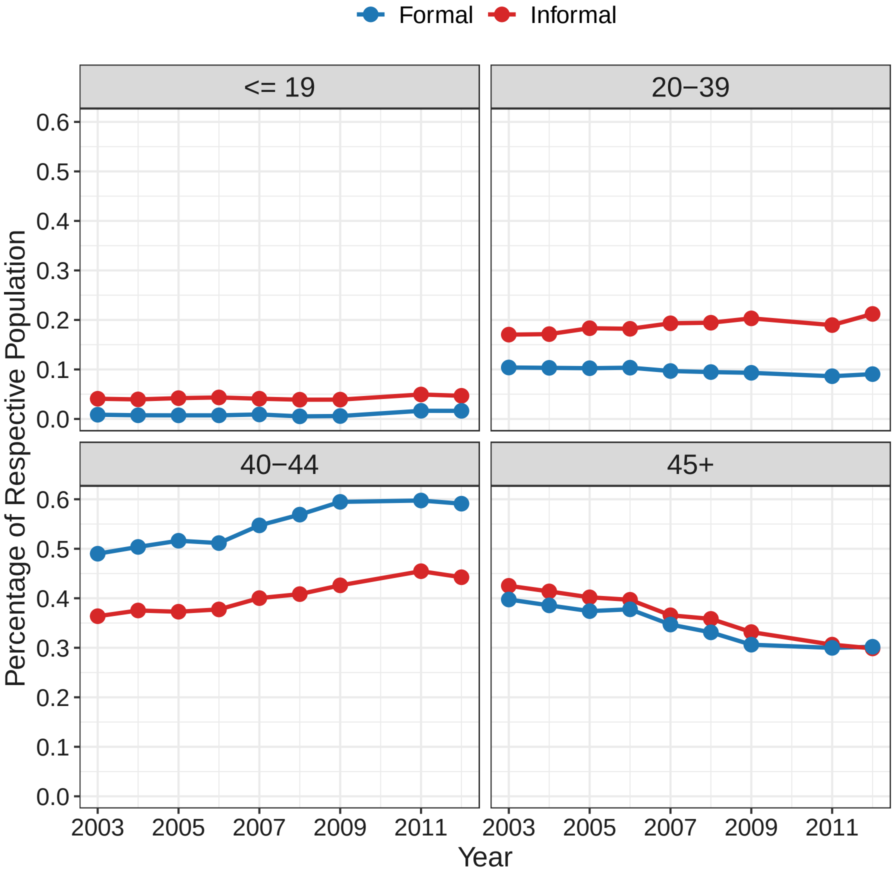


Figure F.2: Share of Workers by Range of Hours Worked (2003–2012)

Notes: Calculations take into account survey weights and exclude self-employed and public, agriculture and domestic sectors, as well as the bottom and top 1% of wage earners in each sector.
Source: PNAD 2003–2012. Use this to return to Section 7.1.

G Estimation Details

This appendix gives more details on the construction of the moments used for estimation and on our SMM routine.

G.1 Moments

Firm Informality and Combining RAIS and ECINF. In order to get an estimate for the degree of firm informality in the economy – which feeds Table 1 and one of the moments used in SMM estimation –, we must combine RAIS and ECINF, as the latter only covers small firms. Although representative of informal firms, the size limitation excludes an important share of formal establishments.

We follow the replication code in Ulyssea (2018) to unify these datasets. The key assumption is that all formal firms with at least one employee in ECINF are in RAIS. This is a natural assumption, given that RAIS is an administrative dataset that is supposed to cover the universe of formal employer firms. Hence, we exclude from ECINF all formal firms with at least one employee and combine its information with RAIS. We keep the sampling weights from ECINF and assign unitary weights to the RAIS data.

Smoothing the Empirical Wage CDFs. Section 6.4 already describes the main moments used. We now detail how we smooth the empirical wage CDFs to be consistent with the continuity of the distribution in the model at all points outside the minimum wage. Our procedure relies on kernel density functions, estimated with the `stats::density()` function in `R`. We use a Gaussian kernel with default bandwidth, 512 nodes and weigh the observations according to their sampling weights in PNAD; as weights must sum to one, we use `weight/sum(weight)`.[67] Finally, we use log wages so that the equally-spaced grid is denser at smaller values.

In the formal sector, we estimate a kernel density ranging from $\log(w_{\min} + \zeta)$, where $\zeta = 0.01$, to the log of the maximum observed value in the data (computed after the sample selection criteria in Section 2). In this way, we exclude the minimum wage, which is the only mass point of the wage distribution in our model.[68] We numerically integrate the density estimates using the Newton-Cotes Trapezoidal rule; call the resulting kernel CDF $\tilde{G}_1(w)$. Letting $\hat{v}_1^G$ be the empirical minimum wage spike in the formal sector, our estimate of the wage CDF is:

$$\hat{G}_1(w) = \hat{v}_1^G + (1 - \hat{v}_1^G)\tilde{G}_1(w)$$

Note that this implicitly assumes that there is no measurement error in the minimum wage

---

[67] When estimating densities only below or above the minimum, we restrict `sum(weight)` to only account for observations with wages below or above $w_{\min}$, respectively.

[68] The boundaries of the missing mass $w_1^*(p_1^*)$, $w_2^*(p_2^*)$, and $w_2^*(p_{2*})$ are not jumps in the wage CDF, but rather kinks. Indeed, Proposition 3 shows that the wage offer densities at these points are not continuous, but still exist.

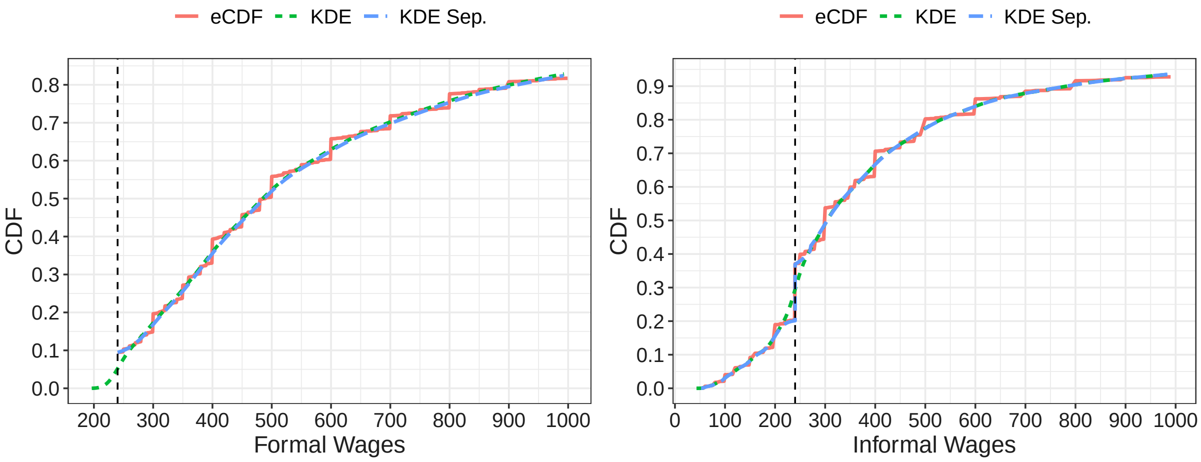
spike: naturally, this affects our parameter estimates, but our focus is on modeling the *existence* of the spike in both sectors. Additionally, the use of the empirical wage CDF, which we denote by $\hat{G}_1^E(w)$, would result in the same minimum wage spike.

The minimum wage spike is an interior point in the wage distribution of the informal sector. Hence, we must estimate two distributions: one below the minimum (from the log of the lowest wage to $\log(w_{\text{min}} - \zeta)$) and one above it (from $\log(w_{\text{min}} + \zeta)$ to the log of the highest wage). Let $\tilde{G}_{2,\text{below}}(w)$ and $\tilde{G}_{2,\text{above}}(w)$ be the resulting kernel CDFs. Our estimate for the wage CDF is:

$$\hat{G}_2(w) = \hat{G}_2^E(w_{\text{min}} - \zeta) \cdot \tilde{G}_{2,\text{below}}(w) + \hat{v}_2^G + (1 - \hat{G}_2^E(w_{\text{min}}))\tilde{G}_{2,\text{above}}(w)$$

In words, we take the estimated CDF conditional on salaries being below/above the minimum wage – $\tilde{G}_{2,\text{below}}(w)$ and $\tilde{G}_{2,\text{above}}(w)$, respectively – and weigh them by the respective share of workers in these regions, as measured by the empirical CDF $\hat{G}_2^E$. We then add to these CDFs the informal minimum wage spike $\hat{v}_2^G$. To assure $\hat{G}_2(w)$ integrates to one, we adjust $\tilde{G}_{2,\text{above}}(w)$ by a kernel CDF estimated on the whole data. Intuitively, for points above a certain distance from w_{min} – which we take to be $w_{\text{min}} + 30$ based on visual evidence on the differences between the two before the adjustment –, these two objects should coincide, and so we add to $\tilde{G}_{2,\text{above}}(w)$ the difference between the kernel CDF estimated on all informal wages and itself, interpolating linearly on the different wage nodes if necessary.

Figure G.1 compares $\hat{G}_1(w)$ and $\hat{G}_2(w)$ to the empirical CDF and to a kernel density integration computed on all wages in each sector. $\hat{G}_1(w)$ and $\hat{G}_2(w)$ only differ significantly from the KDE CDF around the minimum wage, while the eCDF has several mass points due to heaping at round numbers in the PNAD data.



(a) Formal Workers

(b) Informal Workers

Figure G.1: Accepted Wage CDFs in the Data

Notes: This figure compares different methods of estimating the wage CDFs in each sector. “eCDF” is the empirical CDF, while “KDE” is given by estimating kernel densities on all wages and integrating them using trapezoidal integration. “KDE Sep.” corresponds to $\hat{G}_1(w)$ and $\hat{G}_2(w)$. Sample selection as described in Section 2.

Sources: PNAD (2003).

For the moments, we compute quantiles using their definition as the inverse function of the CDFs. The share of workers in the missing mass region in each sector $i = 1,2$ is simply given by $\hat{G}_i(w_{\min} + \Delta^* w_i) - \hat{G}_i(w_{\min} + \zeta)$, using the estimated widths $\Delta^* w_i$ discussed in Section 6.3 and Appendix B. Worker informality is measured directly in the PNAD dataset, and so is not affected by this procedure. Finally, to compute the standard errors required for the weight matrix W_{IV}, we bootstrap the PNAD individual microdata and redo all estimations.

G.2 Estimation

Model Solution. We solve and estimate the model in `Julia` using the random seed `121019`, set at the beginning of the estimation routine.[69] We consider a large number N of firms across different datasets D (so $N \cdot D$ simulated firms in total) and simulate their behavior according to the propositions and assumptions of the model. It will be useful to think of $(N \times D)$ matrices for each object.[70] We take $N = 500\,000$ and $D = 6$.

Specifically, we first generate three sets of $(N \times D)$ independent random draws at the beginning of the routine and keep them fixed during estimation. The first are Uniform$[0,1]$ numbers, which we map into Pareto$(\theta_{\text{scale}}, \theta_{\text{shape}})$ using the `quantile` and `Distributions.Pareto` functions and the Probability Integral Transform. The second are $N(0,\,1)$ draws, which we scale by σ_ε to get the post-entry shocks (in log space). The third are another set of unit uniform numbers, used for the assignment of types.

The entry problem requires computing expected values conditional on the draw of θ. Given the LogNormal shock, one could compute the expected values using Gauss-Hermite quadratures with the appropriate transformations of nodes and weights. However, to save time during estimation, we follow Ulyssea (2018, Appendix F.1) and leverage the normality of $\log(p) \mid \log(\theta)$ to construct a transition matrix in a Tauchen-like manner.

The end product is a set of $(N \times D)$ matrices for each model object (e.g., pre-entry signals, post-entry shocks, effective productivities, sector decisions, Type-A wages if the firm is formal, Type-B labor force sizes if informal, and so on). Expectations with respect to F_i first filter for active firms in sector i and then take the mean. Expectations and the distributions with respect to G_i are similar, but use the labor force sizes ℓ_i as weights for the averages. All moments and other outputs are computed as means or aggregated across datasets.

Weight Matrix. The weight matrix W is given by:

$$W = \frac{1}{2} W_I + \frac{1}{2} \frac{\operatorname{tr}(W_I)}{\operatorname{tr}(W_{IV})} W_{IV},$$

---

[69] Seeds are set both for the `Random` and `NLopt` packages.

[70] Different datasets reduce both the "randomness" of the results and the "choppiness" of the SMM objective function, which can be discontinuous due to the simulation of discrete choices (Altonji, Smith, and Vidangos 2013; Ulyssea 2018). Unlike Bruins et al. (2018) and Ulyssea (2018), we do not smooth the entry decisions.

where all additions and multiplications are element-wise and $\mathrm{tr}(\cdot)$ denotes the trace of the matrix. W_I is the appropriate identity matrix, while $W_{IV} = \mathrm{diag}(\Sigma)^{-1}$ is the inverse of the diagonal of the moments' covariance matrix Σ, which we compute from the data using standard bootstrap procedures. It is well known that the optimal matrix in terms of minimizing asymptotic standard errors in GMM and SMM estimators is Σ^{-1}. However, Altonji and Segal (1996) point out that the off-diagonal elements of this matrix may be poorly estimated, so we take only the diagonal, that is, the variance of the moments, disregarding covariances between them. We then invert this diagonal matrix to get W_{IV}.

W_{IV} gives higher weight to elements that are more precisely estimated, that is, those with a lower variance. However, the widths of missing masses estimated in Appendix B have standard errors that are far larger than the other moments, and so they would receive too little weight. For them to be relevant in the estimation of $\hat{\beta}_{\text{ss}}$, we use W_I to "dilute" the influence of the higher variance. The fraction with the traces of the matrices assures that the elements of W_I and W_{IV} have comparable magnitudes.

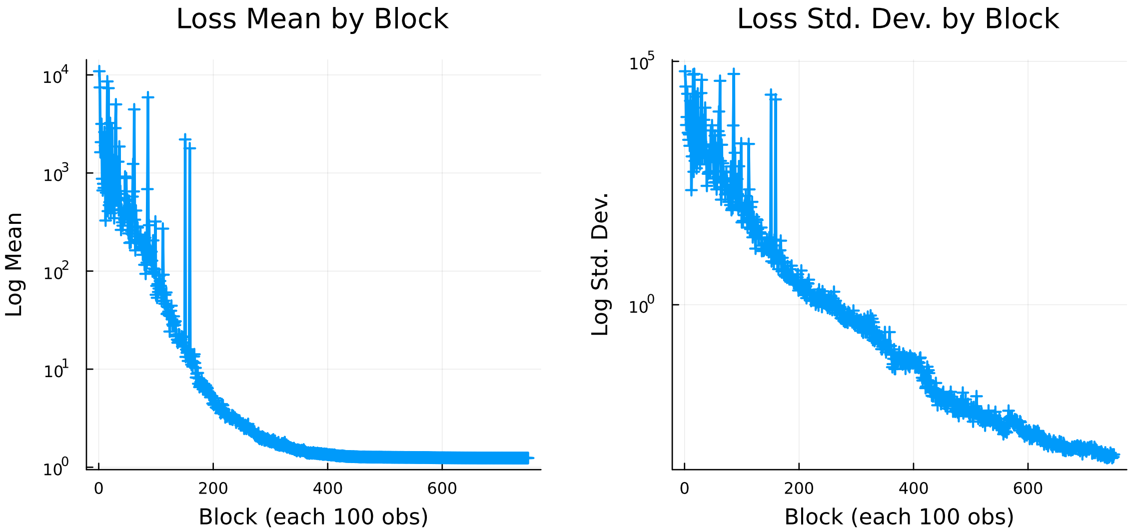
Optimization of Loss Function. We optimize across the space of β_{ss} to minimize the loss function in Equation 16 (Section 6.4). For that, we use the NLopt library. We first use a global algorithm (Improved Stochastic Ranking Evolution Strategy (ISRES)) and then a local algorithm (Subplex) to refine the global solution; both are derivative-free.[71] We inspect the traces of the loss function and of the estimates in the global optimization to verify convergence over iterations. We ensure no final estimate is constrained by pre-specified bounds in the parameter space. For the optimization of the model, we use 75 000 global iterations and 5 000 local iterations to refine the global solution.

In each iteration of the optimization, we take the random draws and adjust them to have the pre-entry signals, post-entry shocks, and type assignments for each simulated firm. We also calculate the transition matrix, which depends on the current "guess" of σ_ε. Given the "guess" of η, we calculate Λ_i according to Section 6.2. We first solve the firms' problem in a pre-specified grid of p, and then interpolate linearly the simulated values of p in this grid. We then use these solutions to find the optimal decision of simulated firms, given their draws of θ, ε, and the type assignments (which are realized after entry). We specify the grid for p in log space and ensure its bounds are sufficient so that all simulated firms can be interpolated.

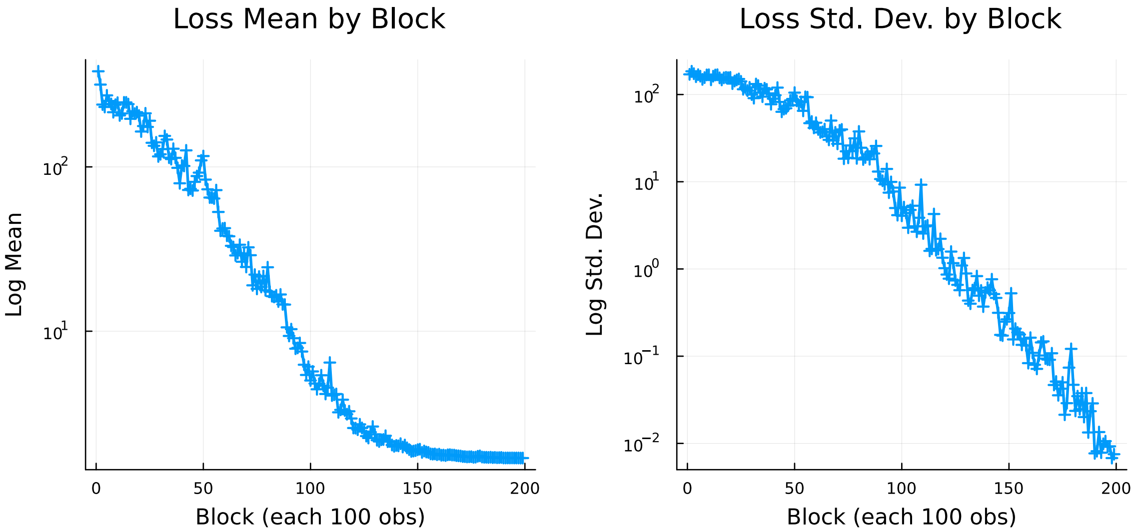
Figure G.2 shows the trace of the loss function in the estimation with and without wage-anchoring, respectively, revealing the convergence of the global algorithm over iterations. The specification without lighthouse effects has a lower number of iterations because it reached the default stopping criterion of the `NLopt` package.

---

[71] We first tested different optimization methods with a lower number of $N \times D$, choosing ISRES because it performed better in terms of loss and of the trace diagnostics. We considered `NLopt` algorithms (Multi-Level Single-Linkage and Controlled Random Search) as well as Simulated Annealing (available as SAMIN in the `Optim` library) with different cooling rates.



(a) Main Model



(b) Model Without Wage-Anchoring

Figure G.2: Trace of the Loss Function – Global Optimization

Notes: The figure shows the evolution of the loss function in the global optimization algorithm. Mean and standard deviations aggregated in blocks of 100 iterations. *Source*: own calculations.

Standard Errors. The matrix used for the sensitivity analysis and the vector of standard errors are computed using the standard SMM asymptotic formulas. As most of our moments come from the PNAD worker data and we simulate firms, we use the sum of labor force sizes across active firms in both sectors in the term of the ratio between data and simulated observations.

G.3 Shapley Values

We now detail the construction of the Shapley values used for the decomposition of Table 5. For a given statistic, we construct intermediate counterfactual distributions by activating subsets of channels: when a channel is “on”, affected firms adopt their counterfactual wages, employment, and sector decision; when “off”, they retain their baseline values. Note, however, that each channel's contribution depends on whether the other channels are active or not: the effect of wage-anchoring on the variance of log wages may depend on whether firms have already switched sectors or exit the market, for example. We then evaluate the statistics on all $2^4 = 16$ possible combinations of active channels, then compute each channel's Shapley value by averaging its marginal contribution across all orderings.[72]

This yields an exact additive decomposition: the Shapley values sum to the total change in the statistic between the baseline and counterfactual equilibria. For the minimum wage spike, we track mass at both the old and new minimum wage levels to capture the migration of the spike across wage bins, and so the change can be marginally smaller due to the baseline mass of workers at the new w_{min}.

To exemplify, consider Table G.1, which shows the values (not the changes relative to baseline) of the statistics following a 60% increase in the real minimum wage, but only turning "on" the channels shown in each row. To get the Shapley value for each channel, we consider all pairs of rows that are identical except that the channel flips from "off" to "on". There are eight such pairs (2^3 combinations of the other three channels). The difference in the statistic between each pair is the marginal contribution of that channel given that specific channel configuration.

We then take a weighted average of the eight marginal contributions, with weights adding to one and depending on the number of other channels that are already on, giving more weight to cases where the channel is the first or last to be turned "on".[73] Table G.2 exemplifies this calculation for the case of the *Wage-Anchoring* channel, with the Shapley values of Table 5 in the last row.

---

[72] Note that this matters for non-linear statistics. If we were to decompose the mean, for example, every ordering would give the same marginal contributions.

[73] The "extreme" cases have weight $1/4$ each, while the other six possibilities have weight $1/12$ each.

Table G.1: Minimum Wage and Wage Inequality Channels for Each On/Off Channel Combination

Channels				MW Spike			Var(log(w))			Informality	Unemployment
Exit	Switch.	Bite	Wage-Anch.	F	I	T	F	I	T		
				10.7	15.1	12.0	100.0	100.0	100.0	29.5	25.1
✓				7.7	15.1	9.9	97.5	100.0	99.8	30.2	26.9
	✓			9.0	15.2	11.0	101.7	101.8	103.3	32.7	29.8
✓	✓			5.5	15.2	8.8	98.2	101.8	103.0	33.5	31.6
		✓		32.1	15.1	27.3	81.1	100.0	88.8	28.2	21.5
✓		✓		30.0	15.1	25.7	77.7	100.0	88.1	28.8	23.3
	✓	✓		32.8	15.2	27.3	79.8	101.8	90.8	31.1	26.2
✓	✓	✓		30.5	15.2	25.6	75.3	101.8	90.0	31.9	28.0
			✓	13.3	16.1	14.2	100.2	96.8	99.3	29.8	25.0
✓			✓	10.4	16.1	12.1	97.7	96.8	99.1	30.4	26.7
	✓		✓	11.9	16.2	13.3	102.0	98.7	102.5	33.0	29.7
✓	✓		✓	8.6	16.1	11.1	98.4	98.7	102.2	33.8	31.4
		✓	✓	34.6	16.1	29.3	81.3	96.8	88.1	28.4	21.4
✓		✓	✓	32.5	16.1	27.8	77.9	96.8	87.5	29.0	23.1
	✓	✓	✓	35.6	16.2	29.5	80.1	98.7	90.1	31.4	26.1
✓	✓	✓	✓	33.3	16.1	27.8	75.5	98.7	89.3	32.1	27.8

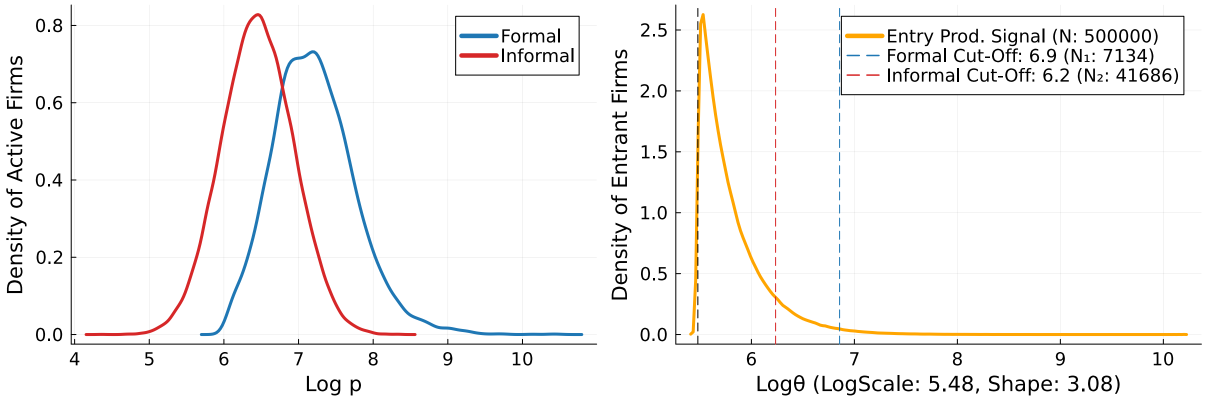
Notes: The table shows the values of summary statistics following an increase of 60% in the real minimum wage $w_{\min}$, considering the existence or not of each of the channels discussed in the main text. The first and last rows correspond to the baseline and counterfactual economy, respectively. "MW Spike" refers to the sum of employment in the old and new minimum wage, so the change is marginally lower than that of Table 4 because of the baseline mass at the new minimum. Variance of wages are calculated based on the distribution of log wages and are normalized by their estimated baseline or 2003 values. Counterfactuals do not consider changes in V^U. "F" = formal sector, "I" = informal sector, "T" = total economy.
Source: model simulations. Use this to return to Section 7.

Table G.2: Marginal Contributions of Wage-Anchoring

				Marginal Changes								Weighted Marginal Changes							
Other Channels				MW Spike			Var(log(w))			Inform.	Unemp.	MW Spike			Var(log(w))			Inform.	Unemp.
Exit	Switch.	Bite	Weight	F	I	T	F	I	T			F	I	T	F	I	T		
			0.25	+2.64	+0.98	+2.15	+0.22	-3.18	-0.72	+0.23	-0.17	+0.66	+0.24	+0.54	+0.05	-0.79	-0.18	+0.06	-0.04
✓			0.08	+2.73	+0.98	+2.21	+0.23	-3.18	-0.73	+0.23	-0.17	+0.23	+0.08	+0.18	+0.02	-0.26	-0.06	+0.02	-0.01
	✓		0.08	+2.95	+0.94	+2.30	+0.27	-3.13	-0.77	+0.24	-0.17	+0.25	+0.08	+0.19	+0.02	-0.26	-0.06	+0.02	-0.01
✓	✓		0.08	+3.06	+0.94	+2.36	+0.29	-3.13	-0.78	+0.24	-0.17	+0.25	+0.08	+0.20	+0.02	-0.26	-0.07	+0.02	-0.01
		✓	0.08	+2.49	+0.98	+2.02	+0.20	-3.18	-0.65	+0.22	-0.17	+0.21	+0.08	+0.17	+0.02	-0.26	-0.05	+0.02	-0.01
✓		✓	0.08	+2.57	+0.98	+2.07	+0.21	-3.18	-0.66	+0.23	-0.17	+0.21	+0.08	+0.17	+0.02	-0.26	-0.06	+0.02	-0.01
	✓	✓	0.08	+2.77	+0.94	+2.15	+0.24	-3.13	-0.69	+0.23	-0.17	+0.23	+0.08	+0.18	+0.02	-0.26	-0.06	+0.02	-0.01
✓	✓	✓	0.25	+2.86	+0.94	+2.21	+0.25	-3.13	-0.70	+0.23	-0.17	+0.71	+0.23	+0.55	+0.06	-0.78	-0.18	+0.06	-0.04
Shapley Value												+2.75	+0.96	+2.18	+0.24	-3.15	-0.71	+0.23	-0.17

Notes: The table shows the values of the changes in summary statistics due to wage-anchoring following an increase of 60% in the real minimum wage $w_{\min}$. Each row corresponds to the change when the wage-anchoring channel is turned "on", given the other (in)active channels. "MW Spike" refers to the sum of employment in the old and new minimum wage, so the change is marginally lower than that of Table 4 because of the baseline mass at the new minimum. Variance of wages are calculated based on the distribution of log wages and are normalized by their estimated baseline or 2003 values. Counterfactuals do not consider changes in V^U. "F" = formal sector, "I" = informal sector, "T" = total economy.
Source: model simulations. Use this to return to Section 7.

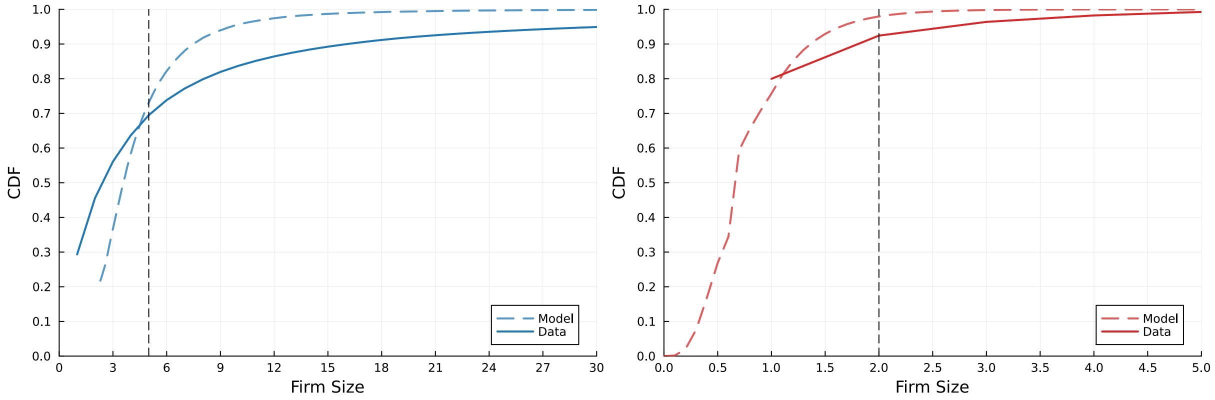
H Additional Model Results and Robustness Tests



(a) Actual Productivity – Active Firms (b) Productivity Signal – Entrant Firms

Figure H.1: Model Productivity Distributions

Notes: The figure shows the densities of the model productivity distributions. The black dashed line in the right panel represents θ_{scale}.
Source: Own calculations based on model simulations. Use this to return to Section 6.5.



(a) Formal Sector (b) Informal Sector

Figure H.2: Firm Size Distributions: Model and Data

Notes: The figure shows the firm size CDFs in the model (dashed lines) and those in the data (solid lines). Dashed black lines at firm sizes of five (formal sector) and two (informal sector).
Source: Own calculations based on model simulations and ECINF, RAIS (2003). Use this to return to Section 6.5.

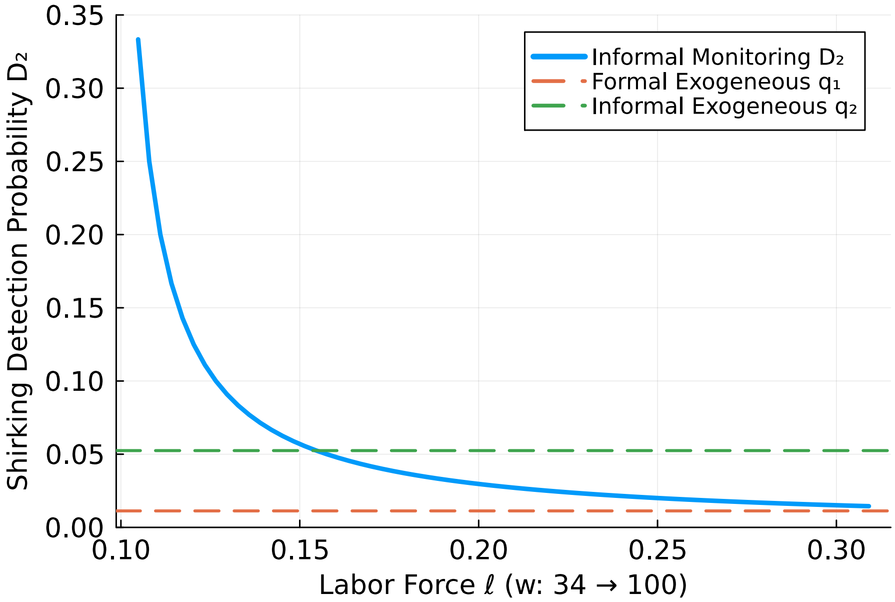


Figure H.3: Implied Monitoring Probabilities $D_2(\ell)$

Notes: The figures shows the implied monitoring probabilities using the parameters in Table D.1 and Proposition 4. *Source*: model simulations. Use this to return to Section D.4.

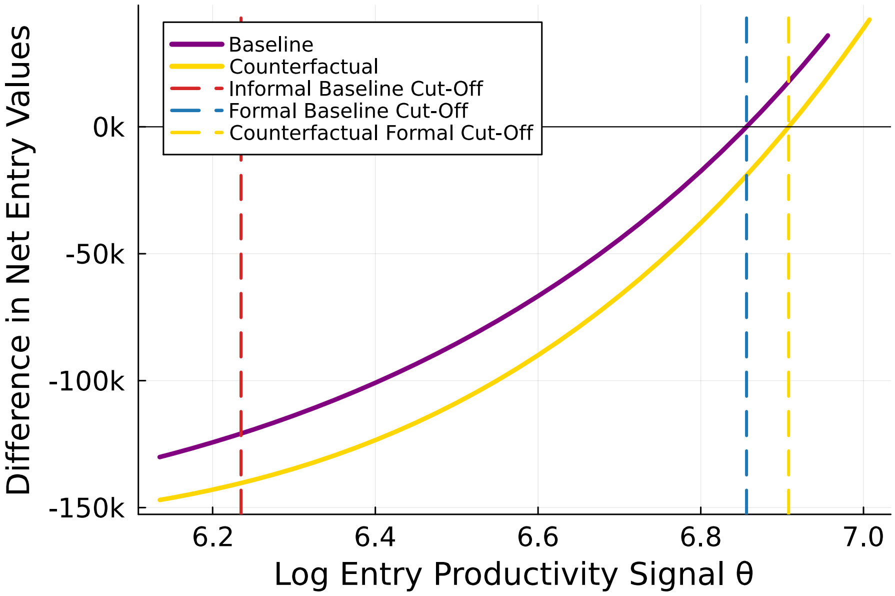


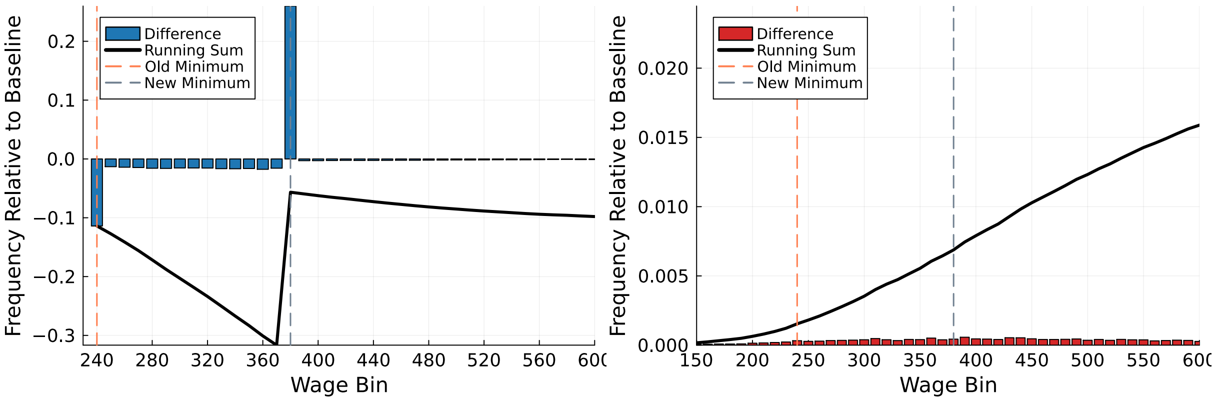
Figure H.4: Differences in Net Entry Values $(\Pi_i(\theta) - E_i)$ Between the Formal and Informal Sectors

Notes: The figure shows the differences in the net entry values $(\Pi_1(\theta) - E_1) - (\Pi_2(\theta) - E_2)$. Counterfactuals do not consider changes in V^U. *Source*: model simulations. Use this to return to Section 7.

Table H.1: Effects of a 60% Increase in the Minimum Wage on Informality, Unemployment, and Inequality

	Main Model			No Wage-Anchoring			Data		
	Base	Δ	Δ MC	Base	Δ	Δ MC	2003	2012	Δ
Informality and Unemployment (%)									
Informality (Firms)	68.5	+5.3	+5.3	68.5	+3.1	+3.1	–	–	–
Informality (Workers)	29.5	+2.6	+2.6	28.9	+1.3	+1.3	25.1	17.0	-8.1
Unemployment	25.1	+2.7	+2.4	25.1	+1.5	+1.8	25.1	14.8	-10.4
Minimum Wage Spike (%)									
Formal	10.7	+22.6	+22.7	9.2	+23.0	+22.9	9.5	13.2	+3.8
Informal	15.1	+1.0	+1.0	0.0	0.0	0.0	16.8	17.8	+1.0
Total	11.9	+16.8	+16.8	6.7	+16.7	+16.7	11.3	14.0	+2.7
Wage Inequality (Formal)									
p_{50}/p_{10}	100.0	-7.4	-7.4	100.0	-7.4	-7.4	100.0	96.2	-3.8
p_{90}/p_{50}	100.0	+0.2	+0.1	100.0	+0.0	+0.0	100.0	97.7	-2.3
p_{75}/p_{25}	100.0	-1.3	-1.3	100.0	-1.5	-1.5	100.0	96.9	-3.1
p_{90}/p_{10}	100.0	-7.2	-7.3	100.0	-7.4	-7.4	100.0	94.0	-6.0
$\text{Var}(\log(w))$	100.0	-24.5	-23.6	100.0	-25.7	-25.6	100.0	74.6	-25.4
Wage Inequality (Informal)									
p_{50}/p_{10}	100.0	+1.6	+1.6	100.0	+0.6	+0.6	100.0	98.5	-1.5
p_{90}/p_{50}	100.0	-1.1	-1.1	100.0	-0.3	-0.3	100.0	97.3	-2.7
p_{75}/p_{25}	100.0	-0.4	-0.4	100.0	+0.4	+0.4	100.0	95.9	-4.1
p_{90}/p_{10}	100.0	+0.5	+0.5	100.0	+0.3	+0.3	100.0	95.9	-4.1
$\text{Var}(\log(w))$	100.0	-1.3	-1.3	100.0	+1.6	+1.5	100.0	82.1	-17.9
Wage Inequality (Total)									
p_{50}/p_{10}	100.0	-1.5	-1.5	100.0	-0.7	-0.7	100.0	96.9	-3.1
p_{90}/p_{50}	100.0	+1.1	+1.1	100.0	+0.1	+0.1	100.0	98.4	-1.6
p_{75}/p_{25}	100.0	-4.9	-4.9	100.0	-4.3	-4.3	100.0	98.0	-2.0
p_{90}/p_{10}	100.0	-0.5	-0.5	100.0	-0.6	-0.6	100.0	95.4	-4.6
$\text{Var}(\log(w))$	100.0	-10.7	-10.4	100.0	-11.3	-11.3	100.0	72.5	-27.5

Notes: The table shows the effect of an increase of 60% in the real minimum wage w_{min}. Changes are in percentage points. “MC” (market changes) considers changes in V^U discussed in Section D.4. Wage inequality measures are calculated based on the distribution of log wages normalized by their estimated baseline or 2003 values. 2012 wages deflated to R$ 2003 using the yearly consumer price index (IPCA). *Source*: model simulations and PNAD (2003, 2012). Use this to return to Section 7.



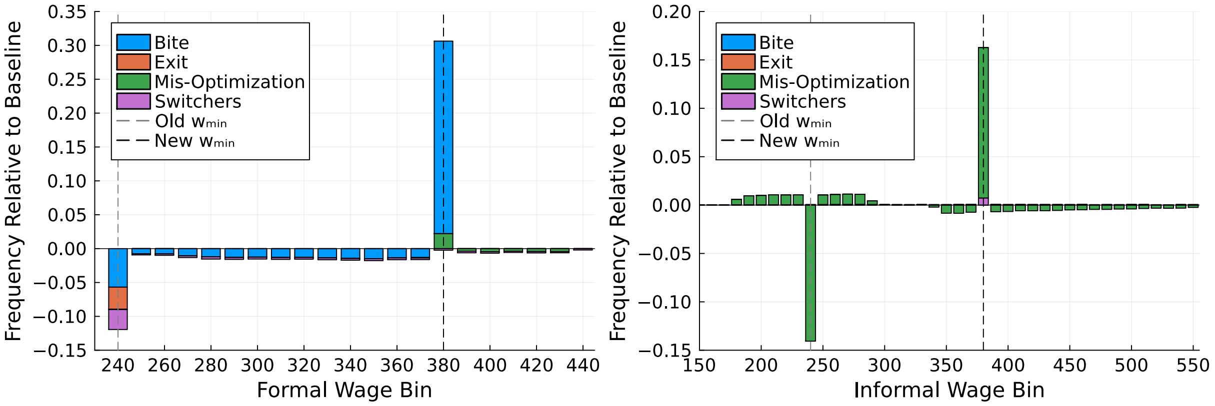
(a) Formal Sector

(b) Informal Sector

Figure H.5: Change in the Frequency Distribution of Wages – No Wage-Anchoring

Notes: The figure shows the differences in R$ 10 wage bins between the new and old accepted wage distributions $G_i(w)$ following a 60% increase in the real minimum wage. Employment is normalized by the pre-treatment total labor in each sector. The black line shows the running sum of the bars until that point. Counterfactuals do not consider changes in V^U.

Source: model simulations. Use this to return to Section 7.



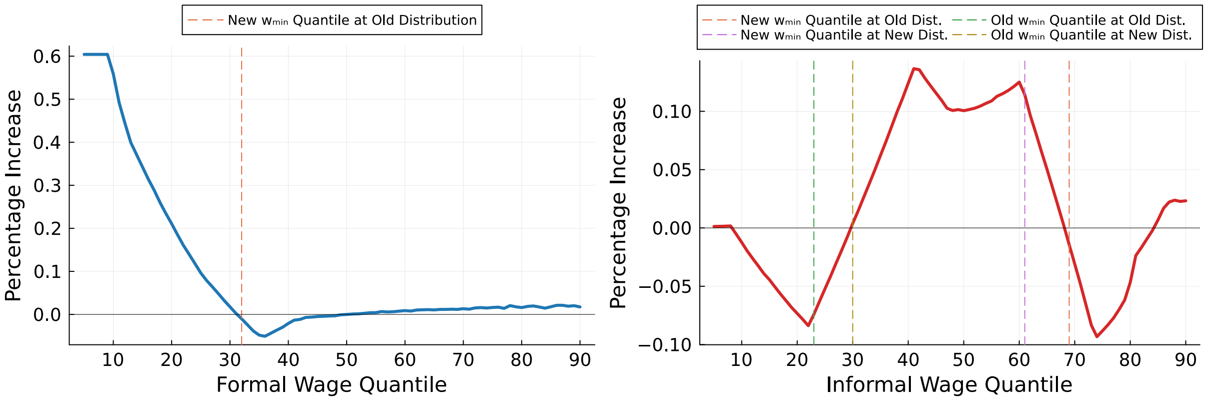
(a) Formal Sector

(b) Informal Sector

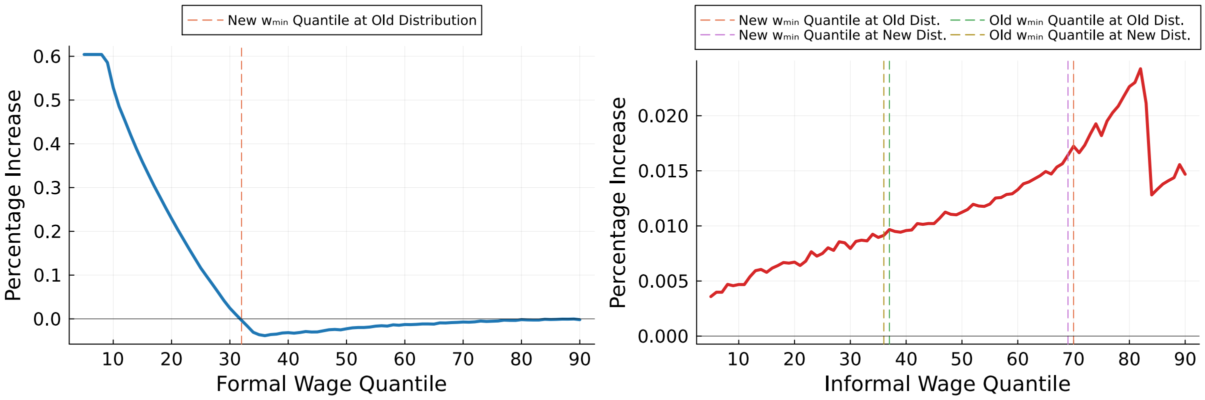
Figure H.6: Decomposition of Change in the Frequency Distribution of Wages – With Wage-Anchoring

Notes: The figures decompose the differences in R$ 10 wage bins between the new and old accepted wage distributions $G_i(w)$ following a 60% increase in the real minimum wage. Employment is normalized by the pre-treatment total labor in each sector. Counterfactuals do not consider changes in V^U.

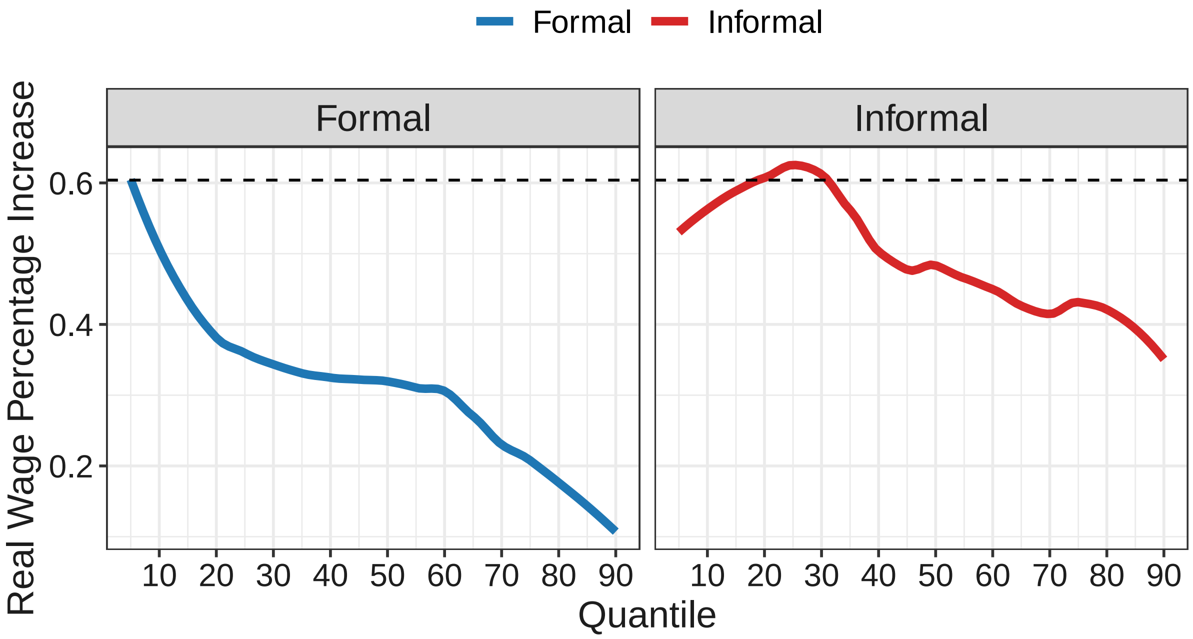
Source: model simulations. Use this to return to Section 7.



(a) Formal Sector, With Wage-Anchoring (b) Informal Sector, With Wage-Anchoring



(c) Formal Sector, No Wage-Anchoring (d) Informal Sector, No Wage-Anchoring



(e) Real Wage Gains, 2003–2012

Figure H.7: Model and Empirical Spillovers

Notes: The figure shows the differences in wages conditional on each quantile. For the model figures, it displays the difference between the counterfactual distribution after a 60% minimum wage increase relative to the baseline model. For the empirical graphs, it shows the difference between the 2012 and 2003 distributions, smoothed using the `geom_smooth(method = "loess", span = 0.5)`. 2012 wages deflated to R$ 2003 using IPCA. Horizontal dashed line in the last graph represents the real minimum wage increase of 60%.
Source: model simulations and PNAD (2003, 2012). Use this to return to Section 7.

H.1 Robustness

Table H.2: Parameter Estimates – Robustness Exercises

Parameter	Description	Logit – With Firm Sizes	Logit – No Firm Sizes	Informal Width 75	Informal Width 47
η	Labor Supply Elasticity	1.29	1.00	0.95	1.02
ξ	Labor Supply Scale	2903.67	1808.04		
σ_1	Formal Share of Type-A Firms	1.00	0.77	0.71	0.81
σ_2	Informal Share of Type-A Firms	0.63	0.64	0.57	0.41
κ_1	Formal Mis-Optimization Cost	0.02	0.01	0.01	0.02
κ_2	Informal Mis-Optimization Cost	0.13	0.08	0.06	0.03
c	Informality Cost Constant	127.77	36.84	56.10	103.69
θ_{shape}	Pareto Shape	3.34	2.42	2.91	3.40
σ_ε	Std. Dev. of Post-Entry Shock	0.61	0.49	0.43	0.47
E_1	Formal Entry Cost	140007.68	236895.78	167154.22	168770.51
E_2	Informal Entry Cost	7569.43	9160.77	8540.94	9381.99

Notes: Parameters are in monthly terms and R$ 2003 values, when applicable.
Source: model simulations based on moments from PNAD, ECINF and RAIS (2003). Use this to return to Section 7.1.

Table H.3: Model Fit – Robustness Exercises

Moment	Source	Data	Logit – With Firm Sizes	Logit – No Firm Sizes	Informal Width 75	Informal Width 47
Wage Moments						
Formal Share $w \leq 251$ (p10)	PNAD	0.101	0.133	0.116	0.119	0.123
Formal Share $w \leq 347$ (p25)	PNAD	0.251	0.261	0.253	0.263	0.263
Formal Share $w \leq 488$ (p50)	PNAD	0.501	0.451	0.478	0.484	0.477
Formal Share $w \leq 793$ (p75)	PNAD	0.750	0.738	0.769	0.753	0.763
Informal Share $w \leq 165$ (p10)	PNAD	0.101	0.124	0.093	0.082	0.091
Informal Share $w \leq 305$ (p50)	PNAD	0.502	0.493	0.492	0.477	0.466
Informal Share $w \leq 474$ (p75)	PNAD	0.750	0.748	0.780	0.793	0.793
Formal MW Spike	PNAD	0.095	0.115	0.105	0.107	0.109
Informal MW Spike	PNAD	0.168	0.136	0.148	0.150	0.150
Formal Share in Missing Mass Region	PNAD	0.031	0.049	0.036	0.037	0.039
Informal Share in Missing Mass Region 105	PNAD	0.210	0.155	0.177		
Informal Share in Missing Mass Region 75	PNAD	0.155			0.123	
Informal Share in Missing Mass Region 47	PNAD	0.088				0.055
Formal Width of Missing Mass 35	PNAD, Bunching	0.146	0.146	0.142	0.144	
Formal Width of Missing Mass 35.3	PNAD, Bunching	0.147				0.147
Informal Width of Missing Mass 105	PNAD, Bunching	0.438	0.445	0.450		
Informal Width of Missing Mass 75	PNAD, Bunching	0.312			0.333	
Informal Width of Missing Mass 47	PNAD, Bunching	0.196				0.224
Share of Informal Workers	PNAD	0.251	0.280	0.287	0.297	0.294
Firm Moments						
Share of Formal Firms with $\ell \leq 5$	RAIS	0.695	0.750			
Share of Formal Firms with $\ell \in (5,10]$	RAIS	0.142	0.199			
Share of Formal Firms with $\ell \in (10,20]$	RAIS	0.084	0.048			
Share of Informal Firms with $\ell \leq 2$	ECINF	0.964	0.964			
Share of Informal Firms	ECINF + RAIS	0.729	0.691	0.687	0.689	0.686

Notes: All moments are in percentage terms. Wage, spike and missing mass shares are with respect to the total of employed workers in that sectors. “Share of Informal Workers” is with respect to the total of employed workers, not considering the unemployed. Width of missing masses are relative to the minimum wage.
Source: model simulations based on moments from PNAD, ECINF and RAIS (2003). Use this to return to Section 7.1.

Table H.4: Effects of a 60% Increase in the Minimum Wage – Robustness Exercises

	Logit – With Firm Sizes			Logit – No Firm Sizes			Informal Width 75			Informal Width 47		
	Base	Δ	Δ MC	Base	Δ	Δ MC	Base	Δ	Δ MC	Base	Δ	Δ MC
Informality and Unemployment (%)												
Informality (Firms)	69.1	+6.9	+6.9	68.7	+3.8	+3.8	68.9	+5.0	+5.0	68.6	+5.7	+5.7
Informality (Workers)	28.0	+2.8	+2.7	28.7	+1.7	+1.7	29.7	+2.4	+2.4	29.5	+2.7	+2.6
Unemployment	25.1	+3.6	+3.0	25.1	+1.7	+2.0	25.1	+2.5	+2.4	25.1	+3.2	+2.4
Minimum Wage Spike (%)												
Formal	11.5	+17.0	+17.0	10.5	+22.5	+22.4	10.7	+23.1	+23.1	10.9	+21.8	+22.0
Informal	13.6	+0.8	+0.8	14.8	+1.7	+1.7	15.0	+0.6	+0.6	15.0	+0.9	+0.8
Total	12.1	+12.6	+12.6	11.7	+16.8	+16.8	11.9	+17.0	+17.0	12.1	+16.1	+16.2
Wage Inequality (Formal)												
p_{50}/p_{10}	100.0	-7.2	-7.2	100.0	-7.4	-7.4	100.0	-7.1	-7.1	100.0	-7.2	-7.4
p_{90}/p_{50}	100.0	-0.0	-0.0	100.0	-0.1	-0.1	100.0	-0.1	-0.1	100.0	-0.2	+0.1
p_{75}/p_{25}	100.0	-1.8	-1.8	100.0	-1.5	-1.5	100.0	-1.8	-1.8	100.0	-2.0	-2.0
p_{90}/p_{10}	100.0	-7.2	-7.2	100.0	-7.5	-7.5	100.0	-7.3	-7.3	100.0	-7.3	-7.3
Var(log(w))	100.0	-29.8	-30.0	100.0	-28.3	-28.3	100.0	-21.7	-21.7	100.0	-25.0	-25.1
Wage Inequality (Informal)												
p_{50}/p_{10}	100.0	+1.1	+1.1	100.0	+0.4	+0.4	100.0	+0.5	+0.5	100.0	-0.6	-0.6
p_{90}/p_{50}	100.0	-1.1	-1.1	100.0	-1.3	-1.3	100.0	-0.3	-0.3	100.0	-0.3	-0.6
p_{75}/p_{25}	100.0	+0.2	+0.2	100.0	-0.8	-0.8	100.0	-0.4	-0.4	100.0	+0.0	+0.4
p_{90}/p_{10}	100.0	0.0	0.0	100.0	-0.9	-0.9	100.0	+0.3	+0.3	100.0	-0.9	-1.2
Var(log(w))	100.0	-2.1	-2.1	100.0	-1.7	-1.7	100.0	+0.4	+0.4	100.0	+0.6	+0.6
Wage Inequality (Total)												
p_{50}/p_{10}	100.0	0.0	-0.4	100.0	-1.8	-1.8	100.0	-1.1	-1.1	100.0	-1.1	-1.1
p_{90}/p_{50}	100.0	+0.3	+0.6	100.0	+0.5	+0.5	100.0	+0.5	+0.5	100.0	+0.7	+0.7
p_{75}/p_{25}	100.0	-5.6	-5.6	100.0	-4.6	-4.6	100.0	-4.9	-4.9	100.0	-4.3	-4.3
p_{90}/p_{10}	100.0	+0.3	+0.3	100.0	-1.3	-1.3	100.0	-0.6	-0.6	100.0	-0.5	-0.5
Var(log(w))	100.0	-10.8	-10.8	100.0	-12.7	-12.7	100.0	-9.6	-9.6	100.0	-9.7	-9.7

Notes: The table shows the effect of an increase of 60% in the real minimum wage $w_{\min}$. Changes are in percentage points. “MC” (market changes) considers changes in V^U discussed in Section D.4. Wage inequality measures are calculated based on the distribution of log wages and are normalized by their estimated baseline or 2003 values. 2012 wages deflated to R$ 2003 using the yearly consumer price index (IPCA). *Source*: model simulations and PNAD (2003, 2012). Use this to return to Section 7.1.

I Sensitivity Analysis

This appendix describes how we implement the sensitivity analysis of Andrews, Gentzkow, and Shapiro (2017) (AGS), which intuitively measures how parameter estimates respond to small changes in the targeted moments. In their words (p. 1559), “our analysis takes as given that a model is identified, and describes the way a specific estimator maps data features into results”.

We conduct two sets of analyses: one in which the model estimates fully adjust to the marginal changes in the moments and another in which the entry decisions are fixed, which allows us to isolate the effects of changes in the moments from general equilibrium effects operating through entry decisions. In all cases, we keep Λ_i and all random draws the same as those in the main model. For the analysis fixing entry, we allow firms with negative post-entry profits to exit the market and abstract from changes in entry costs and the parameters of the productivity distributions, which primarily affect entry decisions. Specifically, we remove these parameters when computing the Jacobian. Finally, since all moments are expressed on the same scale, we report the sensitivity results in the units of the respective parameter.

I.1 Fixing Entry Decisions

To gain intuition, we first analyze the sensitivity fixing entry decisions and abstracting from general equilibrium effects in the form of changes in the entry decisions. Due to the number of moments, we follow AGS’s suggestion and aggregate similar moments, averaging within groups of moments. In the graphs, “(In)Formal Bottom/Middle/Top Third” correspond to the CDF of wages in the bottom/middle/top terciles of the distribution.

Labor Supply Elasticity. As expected, a larger share of workers earning below a wage threshold decreases the estimate for η: a lower η implies more monopsony power and higher markdowns, decreasing wages. At the same time, a higher minimum wage spike and share in the missing mass (MM) region also decreases the estimate for η through the same mechanism: firms would have set wages with higher markdowns, but are constrained by the minimum or fall into the missing mass area. Finally, $\Lambda_1 > \Lambda_2$ and the convexity of our parametrization of $\ell_i(w)$ imply that a lower η is necessary to increase the share of informal workers.

Wage-Anchoring Parameters. The sensitivity results confirm the intuitions in Figure 6: a higher share of Type-A firms is required if the spike decreases or if the share of workers in the MM area increases. At the same time, wage-anchoring costs are directly informed by the widths of the MM region.

Informality Costs. The informality cost constant c in $C(\ell_2) = c\,\ell_2(w)^2$ is more affected by the CDF of informal wages: a higher c raises the cost of informal employment, leading firms to offer lower wages and downsize, increasing the share of workers earning below any given threshold.

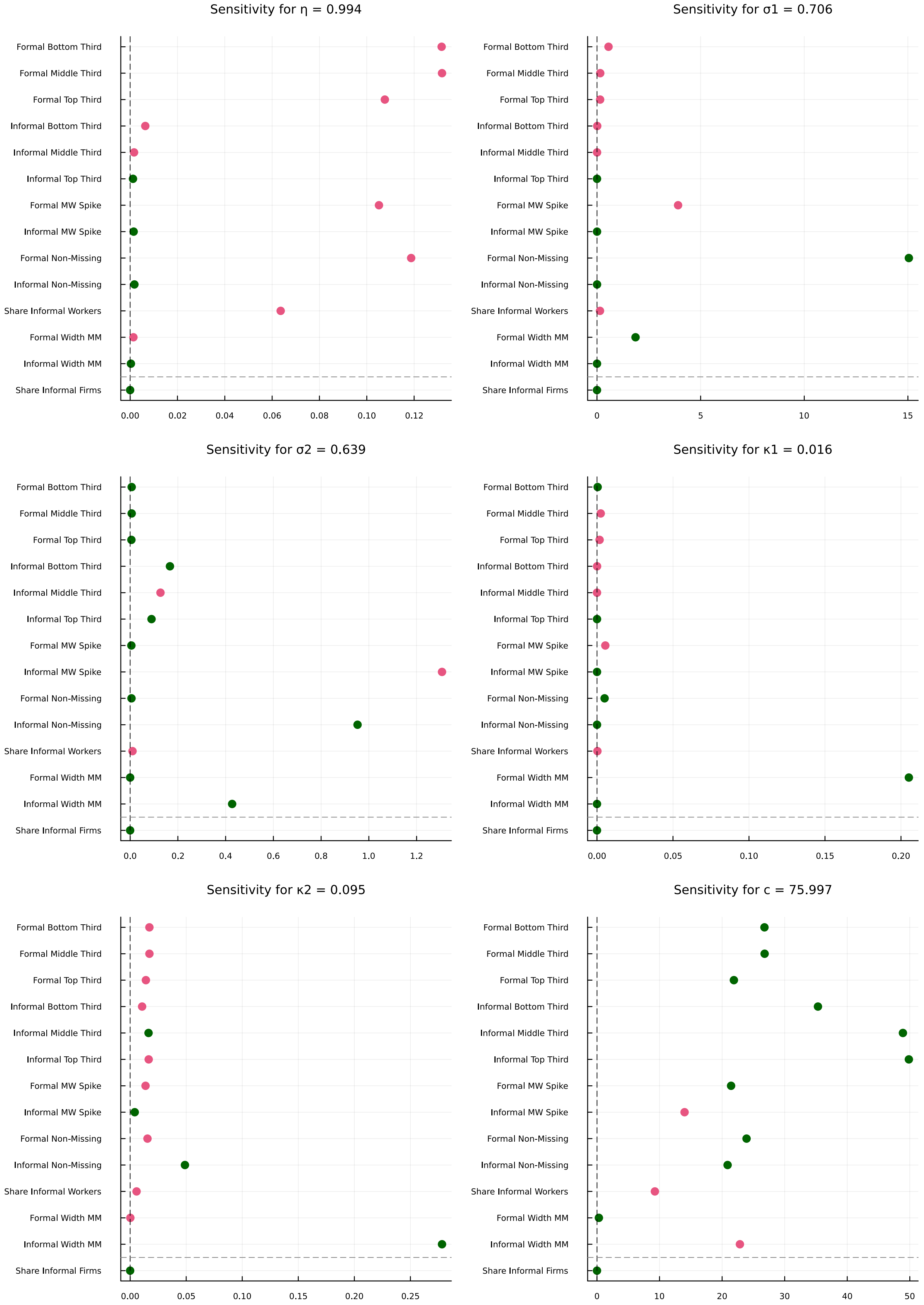


Figure I.1: Sensitivity Analysis – Fixing Entry Decisions

Notes: The figure shows the values of the columns of the sensitivity matrix (moments) for each row (estimated parameter). Green dots represent positive values, while red are negative values. Horizontal axes are in the scale of the respective parameter.
Source: Own calculations based on model simulations. Use this to return to Section 6.5.

I.2 Full Analysis

We now analyze the sensitivity considering changes in entry decisions. We additionally conduct the sensitivity analysis of Adda, Dustmann, and Stevens (2017), measuring how the loss function varies following small changes in the optimal parameters. We fix the calibrated Λ_1 and Λ_2 and keep the same random draws as the baseline model. Figure I.2 and Figure I.3 show the results. Overall, our key parameters appear to be well-identified and sensitive to the moments they should intuitively be.

Labor Supply Elasticity. η is most sensitive to the share of formal workers in the missing mass region and at the minimum wage. At first glance, the opposite signs on the shares of informal firms and workers may appear counterintuitive. However, this is because an increase in η has two effects. First, it makes the formal sector more attractive because of the convexity of $\ell_i(w)$, and so firms with a signal θ that is at the margin of formality now switch sectors – increasing the share of formal workers – and some of them become constrained by w_{min}, which explains the opposite sign of the formal spike relative to the case of fixing entry.

However, a larger η increases profits by increasing firm sizes and reducing markdowns, and so small firms that could not afford the informal entry costs now become active: due to the Pareto productivity assumption, the latter effect dominates the switching to the formal sector in terms of firm informality. As these firms are the smallest in the economy, this effect is not enough to overturn the larger share of formal workers due to firms switching to formality. Finally, some of these new informal firms pay wages in the missing mass region, hence the positive sign.

Figure I.4 illustrates these effects, showing how an increase in η decreases both entry thresholds (moving the solid lines to the dashed ones). As a result, out of 500 000 firms in each simulated dataset, there are around 24 000 new entrants in the informal sector, while 9 000 previously informal firms switch to formality. Figure I.3 shows that the loss function is highly sensitive to changes in η.

Wage-Anchoring Parameters. The intuition is the same as in the fixed entry case: reassuringly, our key parameters are identified by the moments discussed in Figure 6. κ_2 is also influenced by moments in the formal sector due to general equilibrium effects, as a higher share of workers at or near the minimum wage in the formal sector requires more small formal firms, which in turn can be achieved by higher wage-anchoring costs in the informal sector, meaning that informality is less profitable in expectation. Note that, as these parameters are heavily influenced by a small subset of moments, they do not have much impact on the overall loss function in Figure I.3.

Informality Costs. Figure I.2 suggests that c is affected by various moments. Reassuringly, the impact of the share of informal firms is among the largest and carries the expected sign: a lower informality cost is required to increase firm informality.

Productivity Distributions. We start with θ_{shape}. As argued in Section 6.3, it is informed by the top tercile of the formal distribution: a higher θ_{shape} implies more concentrated productivity and wage distributions, such that wages are lower on average. A more concentrated distribution is also required to create relatively larger minimum wage spikes and increase the share of workers in the MM region. The post-entry shock standard deviation σ_ε is sensitive to changes in the bottom and top of the informal distribution (with opposite signs), as well as the share of formal workers at and near the minimum wage, as both require a larger entry uncertainty.

Entry Costs. As with the informality cost, the entry costs are affected by changes in several moments. Interestingly, the signs for E_1 and E_2 are the same for all moments considered, suggesting that they are not separately identifiable. Nonetheless, Figure I.3 shows that the loss function is highly sensitive to changes in the optimal values.

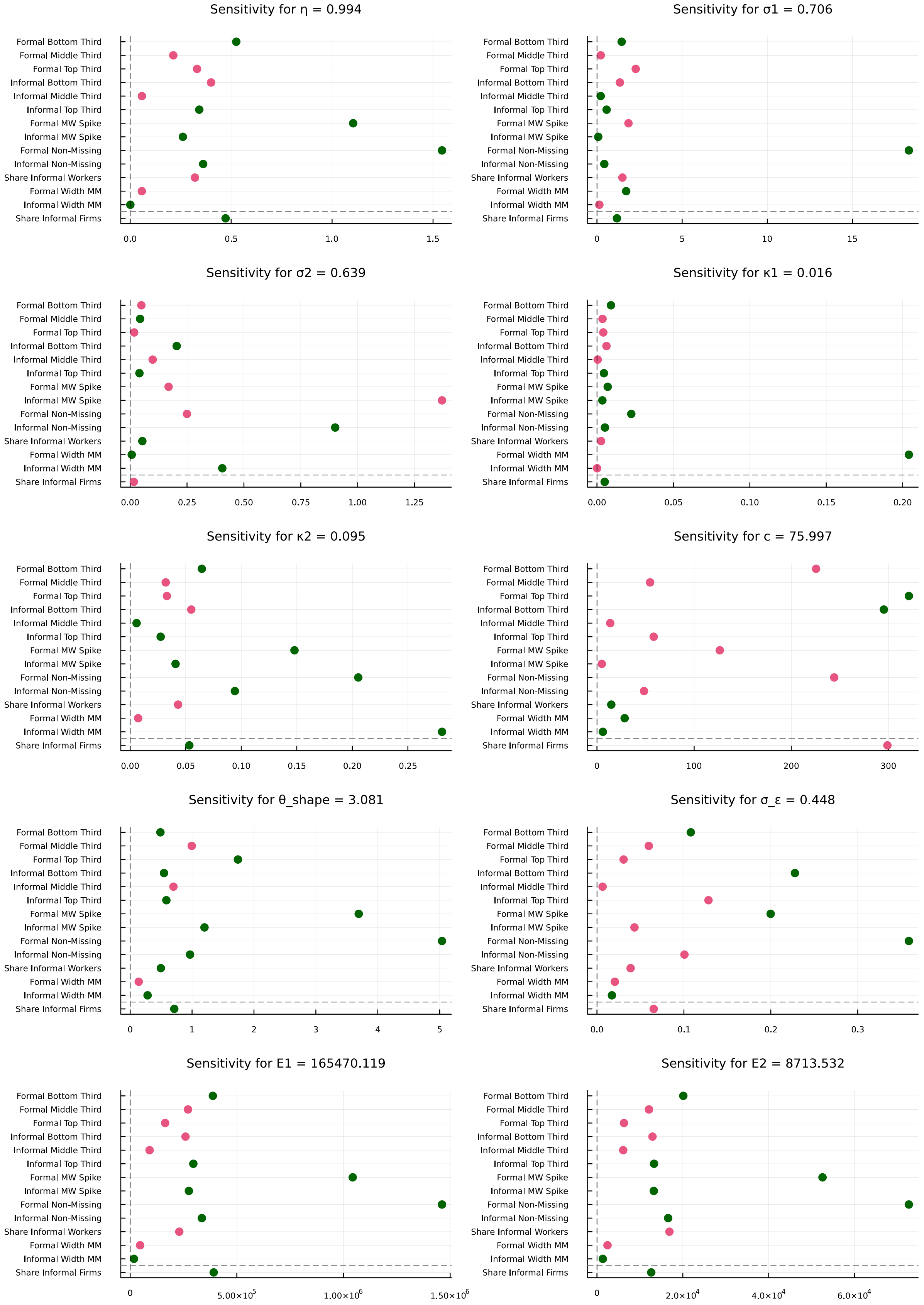


Figure I.2: Sensitivity Analysis

Notes: The figure shows the values of the columns of the sensitivity matrix (moments) for each row (estimated parameter). Green dots represent positive values, while red are negative values. Horizontal axes are in the scale of the respective parameter.
Source: Own calculations based on model simulations. Use this to return to Section 6.5.

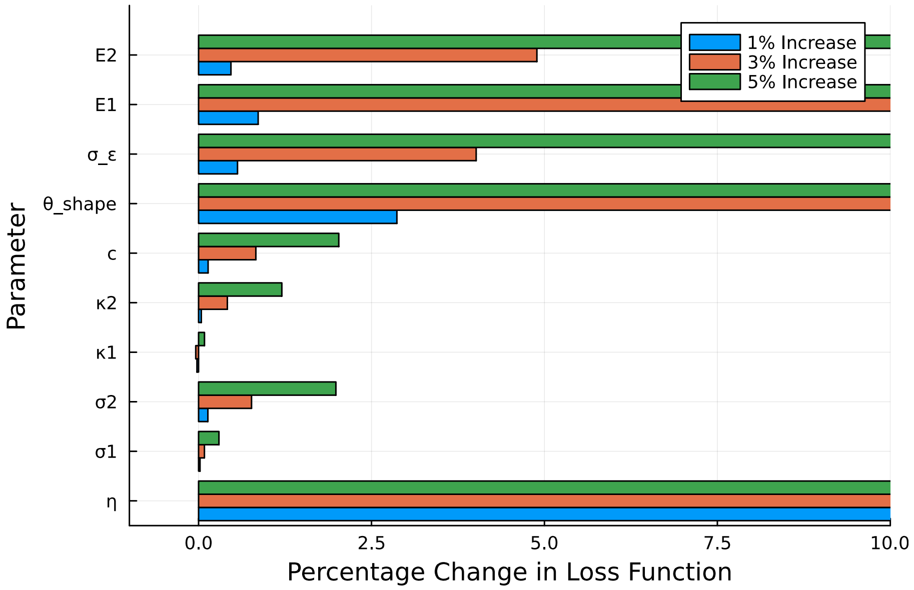


Figure I.3: Changes in Loss Function Following Changes in Parameter Estimates

Notes: The figure shows the effect of an increase in each parameter in the loss function.
Source: Own calculations based on model simulations. Use this to return to Section 6.5.

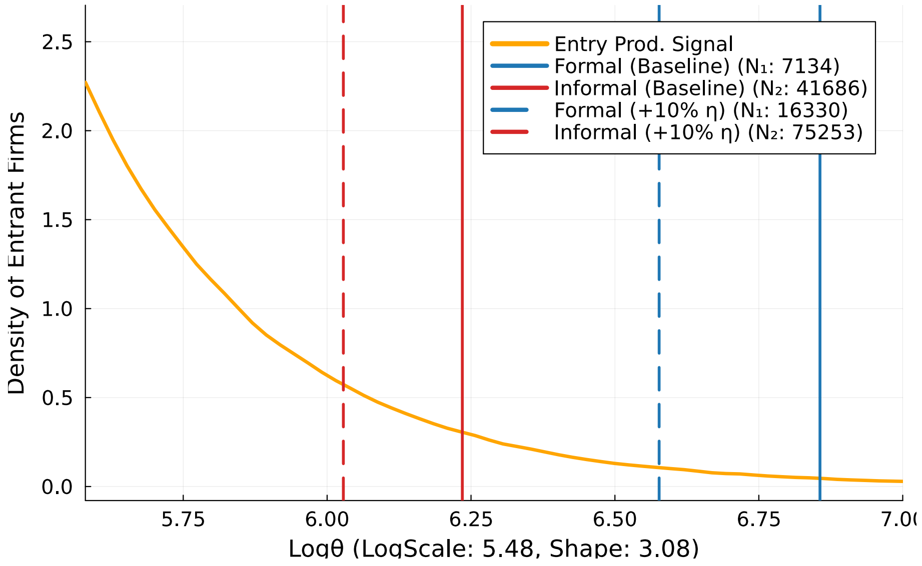


Figure I.4: Effects of an Increase in η in the Entry Decisions of Firms

Notes: The figure shows the density of $\log(\theta)$ and the informal and formal entry thresholds in the main model and when η increases by 10%.
Source: Own calculations based on model simulations. Use this to return to Section 6.5.